Spočítejte primitivní funkci:

1) $\int x^{\frac{3}{2}} + x^{\frac{2}{3}}\,dx = \int x^{\frac{3}{2}}dx + \int x^{\frac{2}{3}}dx = \frac{2}{5}x^{\frac{5}{2}} + \frac{3}{5}x^{\frac{5}{3}} + c$

2) $\int \frac{(1-x)^3}{x}dx = \int \frac{-x^3+3x^2-3x+1}{x}dx = \int -x^2+3x-3+\frac{1}{x}\,dx =$

$-\int x^2dx + \int 3x\,dx - \int 3\,dx + \int \frac{1}{x}dx = -\frac{x^3}{3} + \frac{3x^2}{2} - 3x + \log|x| + c$

$\int 2\cdot\frac{1}{x-1}$

3) $\int \frac{x+1}{x-1}dx = \int \frac{x-1}{x-1} + \frac{2}{x-1}\,dx = \int 1dx + \int \frac{2}{x-1}dx = x + 2\log|x+1| + c$

4) $\int \cot g(x)\,dx = \int \frac{\cos x}{\sin x}dx$ $\quad t = \sin x,\ dt = \cos x\,dx$ $\quad = \int \frac{1}{t}dt = \log|t| = \log|\sin x| + c$

5) $\int x^2 e^{-x}dx$ (f, g') $=$ $\quad f = x^2 \quad g' = e^{-x}$; $\quad f' = 2x \quad g = -e^{-x}$

$= -e^{-x}x^2 + 2\cdot\int e^{-x}x\,dx =$ (g', f) $\quad f = x \quad g' = e^{-x}$; $\quad f' = dx \quad g = -e^{-x}$

$-e^{-x}x^2 + 2\cdot(-e^{-x}\cdot x + e^{-x}) = -e^{-x}x^2 - 2e^{-x}x - 2e^{-x} =$

$-e^{x}\cdot(x^2 - 2x - 2) + c$

6) $\int \frac{2x}{1-x^2}dx = \int \frac{2x}{(1-x)\cdot(1+x)} =$

$\frac{\alpha}{(1-x)} + \frac{\beta}{(1+x)} = \frac{\alpha+\alpha x+\beta-\beta x}{(1-x)\cdot(1+x)}$ $\quad \alpha x - \beta x = 2x$

$\alpha + \beta = 0$

$-\beta x - \beta x = 2x$

$\boxed{\beta = -1 \\ \alpha = 1}$

$= \int \frac{1}{1-x} - \frac{1}{1+x}\,dx = \int \frac{1}{1-x}dx - \int \frac{1}{1+x}dx = -\int \frac{1}{x-1}dx - \int \frac{1}{x+1}dx$

$= -\log|x-1| - \log|x+1| + c$

7) $\int \frac{x+1}{x^2+5x+6}dx =$ $\longrightarrow \frac{x+1}{(x+3)\cdot(x+2)}$ $\quad \frac{\alpha}{(x+3)} + \frac{\beta}{(x+2)}$ $\quad \alpha x + 2\alpha + \beta x + 3\beta$

$2\alpha + 3\beta = 1 \quad 2 - 2\beta + 3\beta = 1 \to \beta = -1$

$\alpha + \beta = 1 \quad \alpha = 1 - \beta \quad \alpha - 1 = 1$

$\alpha = 2$

$\beta = -1$

$\to \int \frac{2}{x+3} - \frac{1}{x+2}dx = 2\cdot\int \frac{1}{x+3}dx - \int \frac{1}{x+2}dx$

$= 2\cdot\log|x+3| - \log|x+2| + c$

$\frac{3x}{(x+2)\cdot(x-1)} = \frac{\alpha}{(x+2)} + \frac{\beta}{(x-1)}$

$\alpha x - \alpha + \beta x + 2\beta = 3x$

8) $\displaystyle\int \frac{x^2 - 2x - 2}{x^2 + x - 2}\,dx = \int \frac{x^2 + x - 2}{x^2 + x - 2} - \frac{3x}{x^2+x-2}\,dx = \int 1\,dx - \int \frac{3x}{x^2+x-2}\,dx =$

$\displaystyle = x - \int \frac{2}{(x+2)} + \frac{1}{(x-1)}\,dx = x - 2\cdot\int \frac{1}{x+2}\,dx + \int \frac{1}{x-1}\,dx =$

$= x - 2\cdot \log|x+2| + \log|x-1| + c$

$\alpha + \beta = 3$

$-\alpha + 2\beta = 0$

$2\beta = \alpha$

$2\beta + \beta = 3$

$\beta = 1$

$\alpha = 2$

9) $\displaystyle\int \underset{f}{x^2}\,\underset{g'}{\cos x}\,dx =$ $f = x^2$, $g' = \cos x$, $df = 2x\,dx$, $g = \sin x$ $\displaystyle = x^2 \cdot \sin x - \int \underset{f}{2x} \cdot \underset{g'}{\sin x}\,dx$ ($df = 2dx$, $g = -\cos x$) $\displaystyle = x^2 \cdot \sin x - \left(2x \cdot (-\cos x) - \int 2 \cdot (-\cos x) \right)$

$= x^2 \cdot \sin x + 2x \cdot \cos x - 2\sin x + c$ $= (x^2 - 2)\sin x + 2x\cos x + c.$

10) $\displaystyle\int \frac{1}{(x+1)\sqrt{x}}\,dx = 2 \cdot \int \frac{1}{t^2+1}\,dt = 2 \cdot \operatorname{arctg}(t) = 2 \cdot \operatorname{arctg}(\sqrt{x}) + c$

$t = x^{\frac{1}{2}}$ $\quad dt = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$x^{\frac{3}{2}} + x^{\frac{1}{2}}$