$$(x+1)^2 > 0 \quad ✓$$

Určete definiční obory a spočtěte derivace:

$$\frac{(x+1)^2}{x^2+1} \geq 0 \rightarrow \quad x^2+1 \geq 0$$
$$x^2 \geq -1 \quad ✓$$

Df: $\mathbb{R}$

1) $$\left(\sqrt{\frac{(x+1)^2}{x^2+1}}\right)' = \left(\left(\frac{(x+1)^2}{x^2+1}\right)^{\frac{1}{2}}\right)' = \frac{1}{2}\cdot\left(\frac{(x+1)^2}{x^2+1}\right)^{-\frac{1}{2}} \cdot \frac{((x+1)^2)'\cdot(x^2+1) - (x+1)^2\cdot(x^2+1)'}{(x^2+1)^2} =$$

$$= \frac{1}{\cancel{2}} \cdot \frac{|x+1|^{-1}}{(x^2+1)^{-\frac{1}{2}}} \cdot \frac{\cancel{2}(x+1)\cdot(x^2+1) - (x+1)^2\cdot \cancel{2}x}{(x^2+1)^2} = \frac{|x+1|^{-1}}{(x^2+1)^{-\frac{1}{2}}} \cdot \frac{(x+1)\cdot(x^2+1) - (x+1)^2\cdot x}{(x^2+1)^2} =$$

$$\frac{x^3+x^2+x+1-x^3-2x^2-x}{(x^2+1)^{\frac{3}{2}}\cdot|x+1|} = \frac{1-x^2}{(x^2+1)^{\frac{3}{2}}\cdot|x+1|}$$

2) $$\left(\ln(\sin(e^x))\right)'$$

$$\sin(e^x) > 0$$

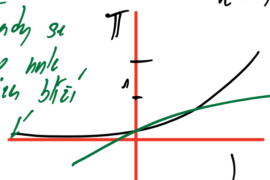
$$\sin(b) > 0 : b \in (0+2k\pi, \pi+2k\pi) \Rightarrow e^x \in (0+2k\pi, \pi+2k\pi)$$

$$e^x > 2k\pi \quad \wedge \quad e^x < \pi + 2k\pi$$
$$\ln(e^x) > \ln(2k\pi) \qquad \ln(e^x) < \ln(\pi+2k\pi)$$
$$x\cdot \ln e > \ln(2k\pi) \qquad x \cdot \ln(e) < \ln(\pi+2k\pi)$$
$$x > \ln(2k\pi) \qquad x < \ln(\pi+2k\pi)$$

Df: $x \in (\ln(2k\pi), \ln(\pi+2k\pi)); k \in \mathbb{Z}$

$$D = (-\infty, \pi) \cup \bigcup_{k=1}^{\infty} (\ln(2\pi), \ln((2k+1)\pi))$$

Tady se to moc jen blíží



$\rightarrow e^x \in (0,\pi)$

$$\ln(\sin(e^x))' = \frac{1}{\sin(e^x)} \cdot (\sin(e^x))' \cdot (e^x)' = \frac{1}{\sin(e^x)} \cdot \cos(e^x) \cdot e^x = \frac{\cos(e^x)}{\sin(e^x)} \cdot e^x = e^x \cdot \cot(e^x)$$

3) $$(x\cos x + \sin(2x^2))' = (x\cdot\cos x)' + (\sin(2x^2))' = x'\cdot\cos x + x\cdot(\cos x)' + (\sin(2x^2))'\cdot(2x^2)' =$$

Df: $\mathbb{R}$

$$= \cos x - x\cdot\sin x + \cos(2x^2)\cdot 4x$$

4) $\left(\frac{3^x+x^3}{3^x-x^3}\right)'$

Df: $3^x - x^3 \neq 0$

$$3^x \neq x^3$$
$$\log_3 3^x \neq \log_3 x^3$$
$$x\log_3 3 \neq 3\log_3 x$$
$$x \neq 3\cdot\log_3 x$$

Df: $\frac{x}{\log_3 x} \neq 3$

$$\left(\frac{3^x+x^3}{3^x-x^3}\right)' = \frac{(3^x+x^3)'\cdot(3^x-x^3) - (3^x+x^3)\cdot(3^x-x^3)'}{(3^x-x^3)^2} = \frac{(3^x\cdot\ln 3+3x^2)\cdot(3^x-x^3)-(3^x+x^3)\cdot(3^x\ln 3-3x^2)}{(3^x-x^3)^2}$$

$$= \frac{\cancel{3^x\cdot 3^x\cdot\ln 3} + 3x^2\cdot 3^x - x^3\cdot 3^x\cdot\ln 3 \cancel{- x^3\cdot 3x^2} \cancel{- 3^x\cdot 3^x\ln 3} + 3^x\cdot 3x^2 - x^3\cdot 3^x\cdot\ln 3 \cancel{+ x^3\cdot 3x^2}}{(3^x-x^2)^2} =$$

$$= \frac{2\cdot 3x^2\cdot 3^x - 2\cdot x^3\cdot 3^x\cdot\ln 3}{(3^x-x^2)^2} = \frac{2\cdot(3^{x+1}\cdot x^2) - 2\cdot(3^x\cdot x^3\cdot\ln 3)}{(3^x-x^2)^2}$$

5) $x^{\sin x}e^{-x^2}$

Df: $\mathbb{R}$

$$(e^{\sin x\ln x})' = e^{\sin x\ln x}\cdot(\sin x\cdot\ln x)' = e^{\sin x\ln x}\cdot\left(\cos x\cdot\ln x+\frac{\sin x}{x}\right) =$$
$$= x^{\sin x}\cdot\left(\cos x\cdot\ln x+\frac{\sin x}{x}\right)$$

$$(x^{\sin x}\cdot e^{-x^2})' = (x^{\sin x})'\cdot e^{-x^2} + x^{\sin x}\cdot(e^{-x^2})' =$$

$f: e^g$

$g: -x^2$

$$e^g\cdot -2x = e^{-x^2}\cdot -2x = -2x\cdot e^{-x^2}$$

$$= x^{\sin x}\cdot\left(\cos x\cdot\ln x+\frac{\sin x}{x}\right)\cdot e^{-x^2} + x^{\sin x}\cdot e^{-x^2}\cdot(-2x)$$

$$= x^{\sin x}\cdot e^{-x^2}\left(\cos x\cdot\ln x+\frac{\sin x}{x}-2x\right)$$