Spočítěte limitu:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots$$

$$\sin x = \sum_{i=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\lim_{x\to 0} \frac{e^x - 1}{x} = \lim_{x\to 0} \frac{1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots - 1}{x} = 1 + \frac{x}{2} + \frac{x^3}{6} + \dots = 1 + \frac{0}{2} + \frac{0}{6} \dots = \underline{\underline{1}}$$

$\frac{0}{0}$!

$$\lim_{x\to\infty} \frac{\sin x}{x} = \frac{x - \frac{x^3}{6} + \frac{x^5}{120}}{x} = 1 - \frac{x^2}{6} + \frac{x^4}{120} = 1 - 0 + 0 \dots = 1$$

$$\sin x = \sum_{i=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\frac{\sin x}{\cos x} \cdot \frac{x}{1} =$$

$$\lim_{x\to 0} \frac{tg(x)}{x} = \frac{\sin x}{\cos x} \cdot \frac{1}{x} = \frac{\sin x}{x \cos x} = = \lim_{x\to 0} \frac{\sin x}{x} \cdot \lim \frac{1}{\cos x} = 1 \cdot 1 = 1$$

$$\lim_{x\to 0} \frac{1 - \cos x}{x^2} = \frac{0}{0} \overset{LH}{=} \lim_{x\to 0} \frac{(1-\cos x)'}{(x^2)'} = \frac{\sin x}{2x} = \frac{1}{2} \cdot \frac{\sin x}{x} = \frac{1}{2} \cdot 1 = \frac{1}{2}$$

$$\lim_{x\to 0} \frac{\ln(x+1)}{x} = \quad g(x) = \ln(x+1) \quad \lim_{x\to 0} \ln(x+1) = 0 \quad \to \lim_{x\to 0} \frac{x}{x} \overset{LH}{=} \lim_{x\to 0} \frac{1}{1} = 1$$

$$\lim_{x\to+\infty} \frac{2x^5 + 7x + 1}{3x^5 + 4x^3 + x^2} = \frac{x^5 \cdot (2 + \frac{7}{x^4} + \frac{1}{x^5})}{x^5 \cdot (3 + \frac{4}{x^2} + \frac{1}{x^3})} = \frac{2}{3}$$

$$(x^2 - x - 2) \cdot (x^2 - x - 2) = x^4 - x^3 - 2x^2 - x^3 + x^2 + 2x - 2x^2 + 2x + 4 = x^4 - 2x^3 - 3x^2 + 4x + 4$$

$$\lim_{x\to 2} \frac{(x^2 - x - 2)^2}{x^3 - 12x + 16} = \frac{(x-2)^2 \cdot (x+1)^2}{(x-2) \cdot (x^2 + 2x - 8)} = \frac{(x-2)^2 \cdot (x+1)^2}{(x-2)^2 \cdot (x+4)} = \frac{(x+1)^2}{x+4} = \frac{6}{4} = \frac{3}{2}$$

$x^3 - 12x + 16 : x - 2 = x^2 + 2x - 8$

$0 + 2x^2$

$2x^2 - 12x$

$0 + 4x$

$-8x + 16$

$0 - 16$

$$\lim_{x\to 3} \frac{x^2 - 5x + 6}{x^2 - 8x + 15} = \frac{(x-2) \cdot \cancel{(x-3)}}{(x-5) \cdot \cancel{(x-3)}} = \frac{x-2}{x-5} = \frac{1}{-2} = -\frac{1}{2}$$

$$(1 + x + 2x + 2x^2) \cdot (1 + 3x) = (2x^2 + 3x + 1)(1 + 3x)$$
$$= 2x^2 + 3x + 1 + 6x^3 + 9x^2 + 3x = 6x^3 + 11x^2 + 6x + 1$$

$$\lim_{x\to 0} \frac{(1+x) \cdot (1+2x) \cdot (1+3x) - 1}{x} = \frac{6x^3 + 11x^2 + 6x + \cancel{1 - 1}}{x} = 6x^2 + 11x + 6 \Rightarrow 6$$

$$\lim_{x\to 0} \frac{\sqrt{1+x} - 1}{x} = \frac{1 + x - 1}{x \cdot \sqrt{1+x} + 1} = \frac{x}{x \sqrt{1+x} + 1} = \frac{1}{\sqrt{1+x} + 1} = \frac{1}{2}$$

$$\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt[3]{1+x}-\sqrt[3]{1-x}} = \frac{\frac{(1+x)-(1-x)}{\sqrt{1+x}+\sqrt{1-x}}}{\frac{(1+x)-(1-x)}{\sqrt[3]{(1+x)^2}+\sqrt[3]{(1+x)\cdot(1-x)}+\sqrt[3]{(1-x)}}} = \frac{\sqrt[3]{(1+x)^2}+\sqrt[3]{(1+x)\cdot(1-x)}+\sqrt[3]{(1-x)}}{\sqrt{1+x}+\sqrt{1-x}} = \frac{3}{2}$$

$$(A-B)\cdot(A^2+AB+B^2) = A^3-B^3$$

$$(A-B)\cdot(A+B) = A^2-B^2$$

$$\lim_{x\to\infty} \sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x} = \frac{x+\sqrt{x+\sqrt{x}}-x}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}} \cdot \frac{\frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x}}} = \frac{\sqrt{\frac{x+\sqrt{x}}{x}}}{\sqrt{\frac{x+\sqrt{x+\sqrt{x}}}{x}}+1} =$$

$$= \frac{\sqrt{1+\frac{\sqrt{x}}{x}}}{\sqrt{1+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x^3}}}}+1} = \frac{1}{2}$$

$$\frac{\sqrt{x}}{x} = \sqrt{\frac{1}{x}} = \frac{1}{\sqrt{x}}$$

$$\lim_{n\to\infty} \ln\left(1+\frac{1}{\sqrt{n}}\right) = \ln\left(\frac{\sqrt{n}+1}{\sqrt{n}}\right) = \ln\left(\frac{n+\sqrt{n}}{n}\right)$$

Nemůžu použít aritmetiku limit, převedu na limitu funkce

$$\lim_{x\to\infty} \ln\left(1+\frac{1}{\sqrt{x}}\right)$$

$f(x) = \ln(x)$

$g(x) = 1+\frac{1}{\sqrt{x}}$

$$\lim_{x\to\infty} 1+\frac{1}{\sqrt{x}} \to 1$$

$$\lim_{x\to 1} \ln(x) = 0$$

$g: P(A,\delta_1)$, $f: U(u,\delta_2)$

Limita složené funkce lze použít, je-li splněno buď:

1) f je spojitá v limitním bodě

2) $\exists \delta: 0<\delta<\delta_1$ t.ž. $u \notin g(P(A,\delta))$

Spočítejte limity:

$$\lim_{x\to\infty} \sqrt{1+\frac{1}{x^2}} = 1$$

$$a^x = e^{\log a^x} = e^{x\log a}$$

$$\lim_{x\to 0} \frac{\ln(\sin x+1)}{x}$$

$f(x) = \frac{\ln(x+1)}{x}$ $\qquad g(x) = \sin x$

$$\lim_{x\to 0} \sin x = 0$$

$$\lim_{x\to 0} \frac{\ln(x+1)}{x} = 1$$

$$\lim_{x\to 0} (x+1)^{\frac{1}{x}} = e^{\frac{1}{x}\log(x+1)}$$

$$\lim_{x\to 0} \frac{\log(x+1)}{x} = 1$$

$$\lim_{x\to 1} e^x = e^1 = e$$

$$\lim_{n\to\infty} \sqrt[n]{n^2+1} = \sqrt[n]{n^2\cdot\left(1+\frac{1}{n^2}\right)} = n^{\frac{2}{n}}\cdot\sqrt[n]{1+\frac{1}{n^2}} = n^0\cdot\sqrt[n]{1+0} = 1\cdot 1 = 1$$

$$\lim_{n\to\infty} \sqrt{n}\cdot\left(\sqrt{n+1}-\sqrt{n-1}\right) = \sqrt{n^2+n}-\sqrt{n^2-n}\cdot\frac{\sqrt{n^2+n}+\sqrt{n^2-n}}{\sqrt{n^2+n}+\sqrt{n^2-n}} = \frac{n^2+n-n^2+n}{\sqrt{n^2+n}+\sqrt{n^2-n}} = \frac{2n}{\sqrt{n^2+n}+\sqrt{n^2-n}} =$$

$$\frac{2n}{\sqrt{n^2\cdot\left(1+\frac{1}{n}\right)}+\sqrt{n^2\cdot\left(1-\frac{1}{n}\right)}} = \frac{2n}{n\cdot\sqrt{1+\frac{1}{n}}+n\cdot\sqrt{1-\frac{1}{n}}} = \frac{2n}{2n} = 1$$

$$(A-B)\cdot(A^2+AB+B^2) = A^3-B^3$$

$$\lim_{n\to 0} \frac{\sqrt[3]{1+x}-1}{x}\cdot\frac{(1+x)^{\frac{2}{3}}+1+(1+x)^{\frac{1}{3}}}{(1+x)^{\frac{2}{3}}+1+(1+x)^{\frac{1}{3}}} = \frac{x}{x\cdot\left((1+x)^{\frac{2}{3}}+1+(1+x)^{\frac{1}{3}}\right)} = \frac{1}{(1+x)^{\frac{2}{3}}+1+(1+x)^{\frac{1}{3}}} = \frac{1}{3}$$

$n \in \mathbb{N}$

$\lim\limits_{x \to 0} \frac{1}{x^n}$ $\begin{cases} n \text{ liché} \\ n \text{ sudé} \end{cases}$

n liché → levé okolí 0 a pravé okolí 0 nemají stejnou limitu, neexistuje

$a^x = e^{\log a^x} = e^{x \log a}$

n sudé → levé i pravé okolí je kladné, tedy má stejnou limitu $+\infty$

$\lim\limits_{x \to 0} \frac{1}{x}$ $\begin{cases} \lim\limits_{x \to 0^+} \frac{1}{x} = +\infty \\ \lim\limits_{x \to 0^-} \frac{1}{x} = -\infty \end{cases}$ Limita tak neexistuje

Základní derivace:

$(konst)' = 0$

$x' = 1$

$x^n = n \cdot x^{n-1}$

$(e^x)' = e^x$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$\log x = \frac{1}{x}$

Derivace: $(\alpha f)' = \alpha f'$

$(f+g)' = f' + g'$

$(f \cdot g)' = f'g + f \cdot g'$

$(f/g)' = \frac{f'g - f \cdot g'}{g^2}$

$f(g))' = f'(g) \cdot g'$

$f \cdot g = \int f' \cdot g + \int f \cdot g'$

Zderivujte:

nebo: $(\sin^2 x)' + (\cos^2 x)' = (\sin x \cdot \sin x)' + (\cos x \cdot \cos x)' = \sin x \cdot \cos x + \cos x \cdot \sin x - \cos x \cdot \sin x - \sin x \cdot \cos x = 0$

$(\sin^2 x + \cos^2 x)' = (\sin^2 x + 1 - \sin^2 x)' = 0$

$(x^x)' = e^{x \log x} = e^{x \log x} \cdot (x \log x)' = x^x \cdot (\log x + x \cdot \frac{1}{x}) = x^x \cdot \log x + x^x = x^x \cdot (\log x + 1)$

$(x^{\log x})' = (e^{\log x^{\log x}})' = (e^{\log x \cdot \log x})' = (e^{\log^2 x})' = x^{\log x} \cdot \frac{2 \log x}{x}$

$f(x) = e^x$

$g(x) = \log x \cdot \log x$

$g'(x) = \frac{\log x}{x} + \frac{\log x}{x}$

$g'(x) = \frac{2 \log x}{x}$

$(\ln(x)^x)' = ?$

$$\sqrt{\frac{(x+1)^2}{x^2+1}}\,' = \left(\left(\frac{(x+1)^2}{x^2+1}\right)^{\frac{1}{2}}\right)' = \frac{1}{2}\cdot\left(\left(\frac{(x+1)^2}{x^2+1}\right)^{-\frac{1}{2}}\right)\cdot\frac{\left((x+1)^2\right)'\cdot(x^2+1) - (x+1)^2\cdot(x^2+1)'}{(x^2+1)^2} =$$

$\cancel{x^3}+x^2+\cancel{x}+1$ $\quad x^2+2x+1 = \cancel{x^3}+2x^2+\cancel{x}$

$$\frac{1}{\cancel{2}}\cdot\frac{|x+1|^{-1}}{(x^2+1)^{-\frac{1}{2}}}\cdot\frac{\cancel{2}\cdot(x+1)\cdot 1\cdot(x^2+1) - (x+1)^2\cdot\cancel{2}x}{(x^2+1)^2} = \frac{|x+1|^{-1}}{(x^2+1)^{-\frac{1}{2}}}\cdot\frac{(x+1)\cdot(x^2+1) - (x+1)^2\cdot x}{(x^2+1)^2} =$$

$$\frac{|x+1|^{-1}}{(x^2+1)^{-\frac{1}{2}}}\cdot\frac{1-x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^{\frac{3}{2}}\cdot|x+1|}$$

$$\ln(\sin(e^x))' = \frac{1}{\sin(e^x)}\cdot\sin(e^x)' = \frac{1}{\sin(e^x)}\cdot\cos(e^x)\cdot e^x = \frac{\cos(e^x)}{\sin(e^x)}\cdot e^x = \operatorname{cotg}(e^x)\cdot e^x$$

$$\left(x\cos x + \sin(2x^2)\right)' = x'\cdot\cos x + x\cdot(\cos x)' + \cos 2x^2\cdot(2x^2)' = \cos x - x\cdot\sin x + \cos 2x^2\cdot 4x$$

vnější x^3
vnitřní $\cos 2x$

$$\left(\cos^3(2x)\right)' = 3\cos^2 2x \cdot (\cos 2x)' = 3\cos^2 2x \cdot (-\sin 2x)\cdot 2 = -6\cos^2 2x \cdot \sin 2x.$$

Musím chodit postupně zvenku dovnitř. Tedy nejdřív derivuji a^3, pak $\cos 2x$, pak $2x$ ... Postupně používám větu o derivaci sl. funkce.

$f(x) = \sin x$ $\quad g(x) = (x-1)^{\frac{1}{2}}$

$f'(x) = \cos x$ $\quad g'(x) = \frac{1}{2\sqrt{x-1}}$

$$\left(\sin\sqrt{x-1}\right)' = \cos\sqrt{x-1}\cdot\frac{1}{2\sqrt{x-1}} = \frac{\cos\sqrt{x-1}}{2\sqrt{x-1}}$$

Použiju substituci a pak zderivuji součin

$\frac{1-x^2}{1+x^2} = d$ $\qquad \left(\sqrt{d}\right)' = \left(d^{\frac{1}{2}}\right)' = \frac{1}{2\sqrt{d}}\cdot d'$

$\to$ je spojitá

$-2x - 2x^3 - 2x + 2x^3$

$$\left(\sqrt{\frac{1-x^2}{1+x^2}}\right)' = \frac{1}{2\sqrt{\frac{1-x^2}{1+x^2}}}\cdot\left(\frac{1-x^2}{1+x^2}\right)' = \frac{1}{2\sqrt{\frac{1-x^2}{1+x^2}}}\cdot\frac{-2x\cdot(1+x^2) - \left((1-x^2)\cdot 2x\right)}{(1+x^2)^2} =$$

$$= \frac{-4x}{2\sqrt{\frac{1-x^2}{1+x^2}}\cdot(1+x^2)^2} = -\frac{2x}{\sqrt{\frac{1-x^2}{1+x^2}}\cdot(1+x^2)^2}$$

2^x

$\log(2)\cdot(x)' = \log 2$

$$e^{\log 2^x} = \left(e^{x\log 2}\right)' = e^{x\log 2}\cdot(x\log 2)' = e^{x\log 2}\cdot\log 2$$

$$\left(x^2\cdot 2^x\right)' = 2x\cdot 2^x + x^2\cdot(2^x)' = 2x2^x + x^2\cdot 2^x\cdot\log 2$$

Spočítejte limitu:

$$\lim_{x\to 1}\frac{x^3 - x^2 - x + 1}{x^3 + x^2 - x - 1} = \frac{0}{0} \overset{L'H}{\Longrightarrow} \lim_{x\to 1}\frac{(x^3 - x^2 - x + 1)'}{(x^3 + x^2 - x - 1)'} = \frac{3x^2 - 2x - 1}{3x^2 + 2x - 1} = \frac{0}{5} = 0$$

$$\lim_{x\to 1}\frac{x^3 + x^2 - x - 1}{x^3 + 2x^2 - x - 2} = \frac{0}{0} \overset{L'H}{\Longrightarrow} \lim_{x\to 1}\frac{(x^3 + x^2 - x - 1)'}{(x^3 + 2x^2 - x - 2)'} = \frac{3x^2 + 2x - 1}{3x^2 + 4x - 1} = \frac{4}{6} = \frac{2}{3}$$

$$\lim_{x\to 0}\frac{\sqrt[3]{1+x} - 1}{x} = \frac{0}{0} \overset{L'H}{\Longrightarrow} \lim_{x\to 0}\frac{\left((1+x)^{\frac{1}{3}} - 1\right)'}{(x)'} = \frac{\frac{1}{3}\cdot(1+x)\cdot(1+x)'}{1} = \frac{1+x}{3} = \frac{1}{3}$$

$\to$ uměli jsme rozepsat pomocí $\sin x = \sum_{i=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$

$$\lim_{x\to 0}\frac{\sin x}{x} = \frac{0}{0} \overset{L'H}{\Longrightarrow} \lim_{x\to 0}\frac{(\sin x)'}{x'} = \frac{\cos x}{1} = 1$$

$$\lim_{x\to 0} x\cdot\sin\frac{1}{x} = \frac{\sin\frac{1}{x}}{\frac{1}{x}} = \frac{0}{0} \overset{L'H}{\Rightarrow} \lim_{x\to 0}\frac{\left(\sin\frac{1}{x}\right)'}{\left(x^{-1}\right)'} = \frac{\cos\frac{1}{x}\cdot\left(-x^{-2}\right)}{-x^{-2}} = \cos\frac{1}{x} = 1$$

—> cos je spojitá (VOLSF

$$\lim_{x\to\infty}\frac{\ln(1+\sqrt{x})}{\ln(1+\sqrt[3]{x})} = \frac{\cancel{\ln 1}\cdot\ln\sqrt{x}}{\cancel{\ln 1}\cdot\ln\sqrt[3]{\ }} = \frac{\frac{1}{2}\cancel{\ln x}}{\frac{1}{3}\cancel{\ln x}} = \frac{\frac{1}{2}}{\frac{1}{3}} = \frac{3}{2}$$

Vyšetřete průběh funkce:

$x^3-12x+16$

$D(f): \mathbb{R}$

Průsečíky os:

y:

$[0;16]$

x:

$x^3-12x+16=0$

$(x-2)\cdot(x^2+2x-8)=0$

$(x-2)\cdot(x-2)\cdot(x+4)=0$

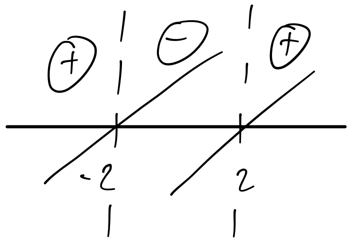
$[2,0]$

$[-4,0]$

$$x^3-12x+16 : x-2 = x^2+2x-8$$
$$\begin{array}{l} -x^3+2x^2 \\ \quad 2x^2-12x \\ -2x^2+4x \\ \quad 0 \quad -8x-16 \\ \qquad +8x+16 \end{array}$$

$x_1=2,\ x_2=-2$

$f' : 3x^2-12 = 3\cdot(x^2-4) = 3\cdot(x-2)\cdot(x+2)$



- kde to je kladný, tam funkce roste, kde záporný, tam klesá!

$f'' = 6x$

- kde je to kladný, tam je konvexní, kde záporný, tam konkávní

Vyšetřete funkci:

$$f(x) = \frac{x^2-1}{x^3-1} : \quad D(f): \mathbb{R} \setminus \{1\}$$

Průsečíky os:

$y: x=0$

$$\frac{0-1}{0-1} = 1 \quad [0,1]$$

$$x: y=0: \frac{x^2-1}{x^3-1} = 0 \quad \begin{cases} [-1,0] \\ [1,0] \ \text{X zakázáno} \end{cases}$$

$$(x-1)\cdot(x+1) = 0$$

$$x = \pm 1$$

$$\lim_{x\to\infty} \frac{x^2-1}{x^3-1} = \lim_{x\to\infty} \frac{x^3\cdot\left(\frac{1}{x}-\frac{1}{x^3}\right)}{x^3\cdot\left(1-\frac{1}{x^3}\right)} = \frac{\frac{1}{x}-\frac{1}{x^3}}{1-\frac{1}{x^3}} = 0$$

$$\lim_{x\to-\infty} = \dots\dots\dots = 0$$

$$\lim_{x\to1} \frac{x^2-1}{x^3-1} \overset{L'H}{=} \lim_{x\to1} \frac{2x}{3x} = \frac{2}{3}$$

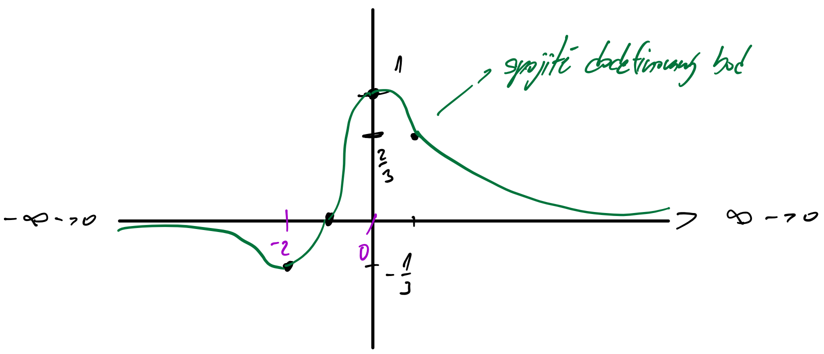
$$f'(x) = \frac{2x\cdot(x^3-1)-\left((x^2-1)\cdot 3x^2\right)}{(x^3-1)^2} = \frac{2x^4-2x-3x^4+3x^2}{(x^3-1)^2}$$

$$(x^3-1)\cdot(x^3-1) = x^6-x^3-x^3+1$$
$$x^6-2x^3+1$$

$$f'(x) = \frac{-x^4+3x^2-2x}{(x-1)^2\cdot(x^2+x+1)^2} = \frac{(x-1)\cdot(-x^3-x^2+2x)}{(x-1)^2\cdot(x^2+x+1)} = \frac{(x-1)^2\cdot(-x^2-2x)}{(x-1)^2\cdot(x^2+x+1)} =$$

$$\frac{-x\cdot(x+2)}{(x^2+x+1)^2} \qquad f'(x)=0 \text{ pro } \begin{matrix} x=-2 \\ x=0 \end{matrix}$$

$$\ominus \ \Big| \ \oplus \ \Big| \ \ominus$$
$$-2 \qquad 0$$



Spočítejte integrál:

$$\int x^2+2x \, dx = \frac{x^3}{3}+x^2+c$$

$$\int (x+2)^2 dx = \begin{matrix} t=x+2 \\ dt=dx \end{matrix} \quad \int t^2 dt = \frac{t^3}{3} = \frac{(x+2)^3}{3}+c$$

$$\int e^x-e^{-x} dx = \int e^x dx - \int e^{-x} dx = e^x - \int e^t - dx = e^x + \int e^t = e^x+e^{-x}+c$$

$$\int \sin x - \cos x \, dx = \int \sin x \, dx - \int \cos dx = -\cos x - \sin x + c$$

$$\int \frac{1+x}{\sqrt{x}}\,dx = \int x^{-\frac{1}{2}} + x^{\frac{1}{2}} = \int x^{-\frac{1}{2}}dx + \int x^{\frac{1}{2}}dx = \boxed{2\cdot x^{\frac{1}{2}} + \frac{2}{3}x^{\frac{3}{2}} + c}$$

$$\int \tan^2 x\,dx = \int \frac{\sin^2 x}{\cos^2 x}\,dx = \int \frac{1-\cos^2 x}{\cos^2 x}\,dx = \int \frac{1}{\cos^2 x} - \int 1\,dx = \boxed{tg\,x - x + c}$$

$$\int x\cdot e^{-x^2} = \quad t = -x^2,\ dt = -2x\,dx \quad = \int \underbrace{e^t\cdot dt}_{e^{-x^2}\cdot(-2x\,dx)}\cdot\left(-\frac{1}{2}\right) = -\frac{1}{2}e^t = \boxed{-\frac{1}{2}e^{-x^2} + c}$$

$$\underbrace{e^{-x^2}\cdot(-2x\,dx)\cdot\left(-\frac{1}{2}\right)}_{e^{-x^2}x\,dx\cdot\cancel{(-2)\left(-\frac{1}{2}\right)}_{1}}$$

$$\int tg\,x\,dx = \int \frac{\sin x}{\cos x}\,dx \quad t = \cos x,\ dt = -\sin x \quad = \int \frac{1}{t}\cdot -1\cdot dt = -\int \frac{1}{t}\,dt = \boxed{-\ln|\cos x| + c}$$

$$f(x)\cdot g(x) = \int f(x)\cdot g'(x)\,dx + \int f'(x)\cdot g(x)\,dx$$

$$\int x\cdot e^x\,dx = \quad \begin{matrix} f(x) = x & g'(x) = e^x \\ f'(x) = 1 & g(x) = e^x \end{matrix} \quad = x\cdot e^x - \int 1\cdot e^x\,dx = \boxed{(x-1)\cdot e^x + c}$$

$$\int x\cdot\sin x\,dx = \quad \begin{matrix} f(x) = x & g'(x) = \sin x \\ f'(x) = 1 & g(x) = -\cos x \end{matrix} \quad = -\cos x\cdot x - \int 1\cdot(-\cos x) = \boxed{-\cos x\cdot x + \sin x + c}$$

$$\int \frac{x}{x^2+1}\,dx \quad \begin{matrix} t = x^2+1 \\ dt = 2x\,dx \end{matrix} \quad = \frac{1}{2}\int \frac{1}{t}\,dt = \frac{1}{2}\ln|t| = \boxed{\frac{1}{2}\ln(x^2+1) + c}$$

$$\int \frac{x^2+1}{x^2-1}\,dx = \int \frac{x^2-1+2}{x^2-1}\,dx = \int \frac{x^2-1}{x^2-1} + \frac{2}{x^2-1}\,dx = \int 1 + \frac{2}{x^2-1}\,dx =$$

$$= \int 1 + \frac{1}{x-1} - \frac{1}{x+1}\,dx = \boxed{x + \ln|x-1| + \ln|x+1| + c}$$

$$\frac{2}{x^2-1} = \frac{\alpha}{(x-1)} + \frac{\beta}{(x+1)} = \frac{\alpha x + \alpha + \beta x - \beta}{x^2-1}$$

$\alpha - \beta = 2 \quad \alpha = 2 + \beta$

$\alpha x + \beta x = 0$

$2x + \beta x + \beta x = 0x$

$2x + 2\beta x = 0x$

$\beta = -1$

$\alpha = 1$

$$\int \frac{x-2}{(x-1)^2}\,dx =$$

$$\frac{x+2}{(x-1)^2} = \frac{\alpha}{(x-1)} + \frac{\beta}{(x-1)^2} = \frac{\alpha x - \alpha + \beta}{(x-1)^2}$$

$\beta - \alpha = -2$

$\alpha x = 1$

$\alpha = 1$

$\beta = -1$

$$= \int \frac{1}{(x-1)} - \frac{1}{(x-1)^2} = \int \frac{1}{x-1}\,dx - \int \frac{1}{(x-1)^2}\,dx =$$

$t = x-1$

$dt = dx$

$$= \ln|x| - \int t^{-2}\,dt = \ln|x| + \frac{1}{t} = \boxed{\ln|x| + \frac{1}{x-1} + c}$$

substituce a otočení znaménka, protože $(t^{-1})' = -t^{-2}$, já chci t^{-2}

$$\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx \quad \begin{matrix} t = e^x + e^{-x} \\ dt = e^x - e^{-x}\,dx \end{matrix} = \int \frac{1}{t}\,dt = \ln|t| = \boxed{\ln|e^x + e^{-x}|}$$

$$\int \ln^2 x\,dx = \int \underset{f}{\ln^2 x} \cdot \underset{g'}{1}\,dx \quad \begin{matrix} f = \ln^2 x & g' = 1 \\ f' = 2 \cdot \log x \cdot \frac{1}{x} & g = x \end{matrix} = \ln^2 x \cdot x - \int \frac{2 \cdot \ln x}{x} \cdot x = \ln^2 x \cdot x - 2 \cdot \int \ln x =$$

$$\ln^2 x \cdot x - 2x(\ln x - 1) = \boxed{x \cdot (\ln^2 x - 2\ln x + 2) + C}$$