$$\int_{\pi/4}^{\pi/3}\left(\left(\cos x\right)\log(\sin x)+(\cos x)^{-2}\right)dx ?$$

$t = \sin x$
$dt = \cos x$

$$\cos x\cdot\log(\sin x)+\frac{1}{\cos^2 x} = \int_{\pi/4}^{\pi/3}\cos x\cdot\log(\sin x)dx + [tg\, x]_{\pi/4}^{\pi/3} =$$

$$= \int_{\pi/4}^{\pi/3}\log t\cdot dt + [tg\, x]_{\pi/4}^{\pi/3} = [t\cdot(\log t - 1)]_{\frac{\sqrt{2}}{2}}^{\frac{\sqrt{3}}{2}} + [tg\, x]_{\pi/4}^{\pi/3} =$$

$$= \frac{\sqrt{3}}{2}\cdot\log\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\log\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} + \sqrt{3} - 1 =$$

$\sin 0 = 0$
$\sin\frac{\pi}{2} = 1$
$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$
$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$$[\sin x\cdot\log(\sin x) - \sin x] + [tg\, x]$$

$tg\,\frac{\pi}{4} = 1$
$tg\,\frac{\pi}{3} = \sqrt{3}$

$$\int\frac{1}{1+x^2} = \arctan x \quad \to (\arctan x)' = \frac{1}{1+x^2}$$

$t = \arctan x$
$dt = \frac{1}{1+x^2}$

$$= \int_1^{\sqrt{3}}\frac{\log(\arctan x)}{1+x^2}dx \; ?$$

$$= \int_1^{\sqrt{3}}\log(t)\,dt = [t\cdot\log t - t]_{\cancel{x} = \frac{\pi}{4}}^{\cancel{\sqrt{3}} = \frac{\pi}{3}}$$

$$= [\arctan x\cdot\log(\arctan x) - \arctan x]_1^{\sqrt{3}}$$

$$= \frac{\pi}{3}\cdot\log\frac{\pi}{3} - \frac{\pi}{3} - \frac{\pi}{4}\cdot\log\frac{\pi}{4} + \frac{\pi}{4}$$

Najděte lokální a globální extrémy:

$f(x) := |x-2|^3 e^x$

$x < 2: \; f(x) := (-x+2)^3 e^x$

$x \geq 2: \; f(x) := (x-2)^3 e^x$

$x < 2: \; f'(x) = -3\cdot(-x+2)^2\cdot e^x + (-x+2)^3 e^x$

$x \geq 2: \; f'(x) = 3\cdot(x-2)^2\cdot e^x + (x-2)^3 e^x$

$x < 2: \; f'(x) = (-x+2)^2\cdot e^x\cdot(-x-1).$

$x \geq 2: \; f'(x) = (x-2)^2\cdot e^x\cdot(x+1).$

$+$ s y: $y = 2^3\cdot e^0 = 2^3 = 8$ $[0,8]$

$+$ s x: $|x-2|^3\cdot e^x = 0$ $[2,0]$

$e^x = 0 \lor |x-2|^3 = 0$

$e^x \neq 0$ na celém def. oboru

$|x-2|^3 = 0$

$x = 2$

$2-x = t$

$-dx = dt$

$((-x+2)^3)' = 3t^2 = -3(2-x)^2$

$x < 2$:

$f'(x) < 0: \; -x-1 < 0$

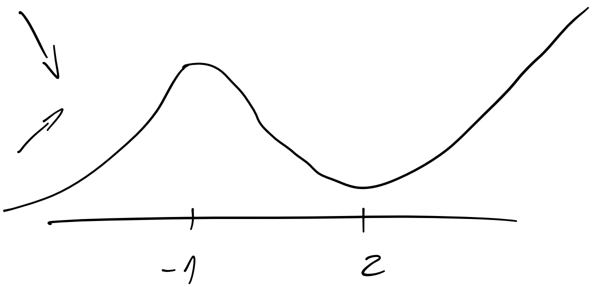
$x > -1$

$f'(x) > 0: \; -x-1 > 0$

$x < -1$

$f'(x) < 0: (-1,2)$ ↘

$f'(x) > 0: (-\infty,-1)$ ↗



$x \geq 2: \; f'(x) < 0: \; x+1 < 0$

$x < -1$

$f'(x) > 0: \; x+1 > 0$

$x > 1$

$f'(x) < 0: \emptyset$

$f'(x) > 0: (2,+\infty)$ ↗

Lokální minimum je pro $x = 2$, lokální maximum je pro $x = -1$.

To je zároveň globální min.

pouze lokální min

$\frac{1}{e^x}$

$\overset{AL}{=} \lim_{x\to-\infty} |x-2|^3 \cdot \lim_{x\to-\infty} e^x \overset{AL}{=} = +\infty \cdot 0 = 0$

$\lim_{x\to-\infty} (-x+2)^3 = (-x+2)\cdot(2-x)^2 = (-x+2)\cdot(4-4x+x^2) =$

$-4x+4x^2-x^3+8-8x+2x^2 =$

$-x^3+6x^2-12x+8$

Spočítejte objem $V(1,2,\log x)$

$$f = \log x \qquad g = x\cdot\log x - x$$
$$f' = \frac{1}{x} \qquad g' = \log x$$

$$\int \log x - 1 = \int \log x - \int 1$$
$$x\cdot\log x - x - x$$

$$\int \log^2 x = \int \log x\cdot\log x = \log x\cdot(x\cdot\log x - x) - \int (x\cdot\log x - x)\cdot\frac{1}{x}$$
$$= \log^2 x\cdot x - \log x\cdot x - (x\cdot\log x - x - x)$$
$$= \log^2 x\cdot x - 2\log x\cdot x + 2x$$

$$V = \pi\int_1^2 \log^2 x = \pi\left(2\cdot\log^2 2 - 4\log 2 + 4 - 0 + 0 - 2\right) = \pi\left(2\log^2 2 - 4\log 2 + 2\right)$$

$$t = \sin x$$
$$dt = \cos x$$

$$\int \cos x\cdot\log(\sin x) + (\cos x)^{-2} = \int \cos x\cdot\log(\sin x) + \int\frac{1}{\cos^2 x} = \int \log(t)\,dt + \operatorname{tg} x =$$
$$\sin x\cdot\log(\sin x) - \sin x + \operatorname{tg} x$$

$$t = \arctan x$$
$$dt = \frac{1}{1+x^2}$$

$$\int \frac{\log(\arctan x)}{1+x^2} = \int \log(t)\,dt = \arctan x\cdot\log(\arctan x) - \arctan x$$

$$\int \frac{1+x}{\sqrt{x}} = \int x^{-\frac{1}{2}} + \int x^{\frac{1}{2}} = 2x^{\frac{1}{2}} + \frac{2}{3}x^{\frac{3}{2}} + C$$

$$t = -x^2$$
$$dt = -2x\,dx$$

$$\int x\cdot e^{-x^2} = -\frac{1}{2}\int e^t\,dt = -\frac{1}{2}e^t = -\frac{1}{2}\cdot e^{-x^2} + C$$

$$f = x \qquad g = -\cos x$$
$$f' = 1 \qquad g' = \sin x$$

$$\int x\cdot\sin x = x\cdot(-\cos x) - \int 1\cdot(-\cos x) = x\cdot(-\cos x) + \sin x$$

$$n^{\sin \frac{n\pi}{2}}$$

$$\sin\frac{\pi}{4}, \quad \sin\frac{\pi}{2}, \quad \sin\frac{3\pi}{4}, \quad \sin\pi$$

$$\begin{array}{cccc} \| & \| & \| & \| \\ \frac{\sqrt{2}}{2} & 1 & \frac{\sqrt{2}}{2} & 0 \end{array}$$

n

$$\limsup = n^{\frac{\sqrt{2}}{2}} = +\infty$$

$$\liminf = n^{0} = 1$$