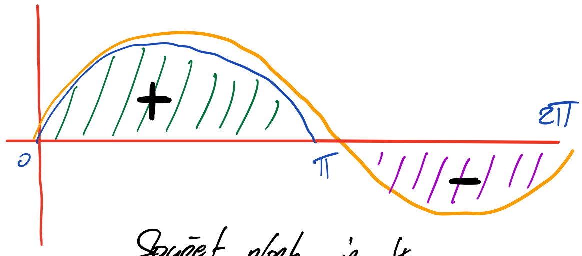
a) $\int \sin x \, dx = -\cos x + C$

b) $\int_0^{\pi} \sin x \, dx = \left[-\cos x\right]_{x=0}^{x=\pi} = -(\cos\pi - \cos 0) = 2$

c) $\int_0^{2\pi} \sin x \, dx = \left[-\cos x\right]_{x=0}^{x=2\pi} = -(\cos 2\pi - \cos 0) = 0$



Součet ploch je 4, musím to ale odečíst

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

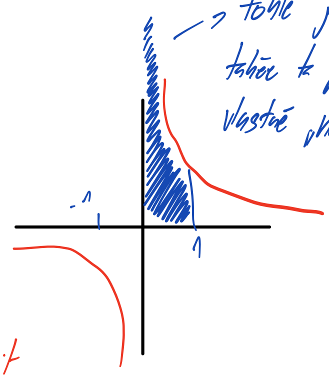
a) $\int \frac{1}{x} dx = \ln|x| + C$

b) $\int_1^2 \frac{1}{x} dx = \left[\ln|x|\right]_{x=1}^{x=2} = \ln 2 - \ln 1 = \ln 2$

c) $\int_{-2}^{-1} \frac{1}{x} dx = \left[\ln|x|\right]_{x=-2}^{x=-1} = \ln 1 - \ln 2 = -\ln 2$

d) $\int_0^1 \frac{1}{x} dx = \left[\ln|x|\right]_{x=0}^{x=1} = \ln 1 - \ln 0 \simeq 1-(-\infty) \simeq +\infty$

tohle jde do nekonečna, takže ta plocha je vlastně „nekonečná"



e) $\int_{-1}^{1} \frac{1}{x} dx = \int_{-1}^{0} \frac{1}{x} dx + \int_0^1 \frac{1}{x} dx = -\infty + \infty$ = což ale nemůžu označit jako 0. Protože $\infty - \infty \neq 0$

– jen v tomhle případě jde o dvě identicky rostoucí nekonečna.

není spojitá na celém intervalu (v 0)

dokonce není v 0 ani definovaná

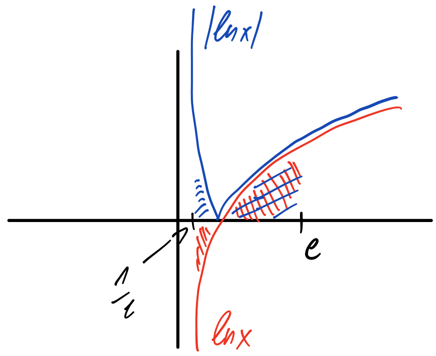
Počítej!

neznám triviální funkci, takže ji rozdělím

a) $\int_{-1}^{1} |x|\,dx = \int_{-1}^{0} -x\,dx + \int_0^1 x\,dx = \left[-\frac{x^2}{2}\right]_{x=-1}^{x=0} + \left[\frac{x^2}{2}\right]_{x=0}^{x=1} =$

$\int_{-1}^{0} |x|dx + \int_0^1 |x|dx$

$= 0 + \frac{1}{2} + \frac{1}{2} + 0 = 1$



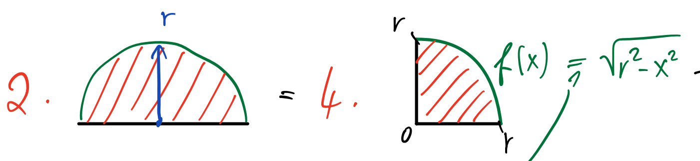
b) $\int_{1/e}^{e} \ln x \, dx = \left[x \cdot (\ln x - 1)\right]_{x=1/e}^{x=e} = 0 + \frac{2}{e}$

c) $$\int_{1/e}^{e} |\ln x|\, dx = \int_{1/e}^{1} \underbrace{|\ln x|}_{-\ln x}\, dx + \int_{1}^{e} \underbrace{|\ln x|}_{\ln x}\, dx = -\Big[x\cdot(\ln x - 1)\Big]_{x=1/e}^{1} + \Big[x\cdot(\ln x - 1)\Big]_{x=1}^{x=e} =$$

rozdělím to tam, kde $\ln x = 0$ tedy $x = 1$

$$= 1 - \frac{2}{e} + 0 + 1 = 2 - \frac{2}{e}$$

Spočtěte plochu kruhu pomocí integrálu!



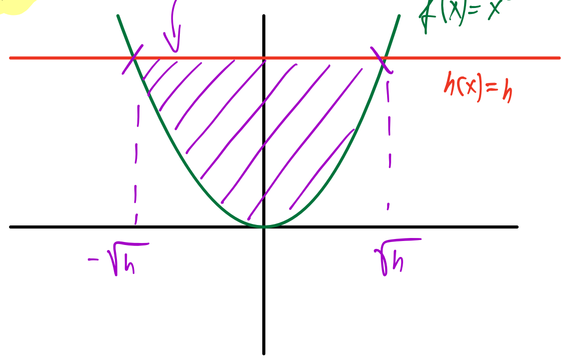
$x^2 + y^2 = r^2$

$y^2 = r^2 - x^2 \quad y = \sqrt{r^2 - x^2}$

$+1$ zlehčení

$$S = 4\cdot\int_0^{1} \sqrt{1-x^2}\, dx = 4\cdot\left[\frac{1}{2}\left(x\cdot\sqrt{1-x^2} + \arcsin x\right)\right]_{x=0}^{x=1} = 2\cdot\left(\frac{\pi}{2} - 0\right) = \pi$$

Nechť mám parabolu a přímku h. Urči plochu mezi parabolou a přímkou



$$\int_{-\sqrt{h}}^{\sqrt{h}} h - x^2\, dx = h\cdot\int_{-\sqrt{h}}^{\sqrt{h}} 1\, dx - \int_{-\sqrt{h}}^{\sqrt{h}} x^2\, dx =$$

$$2h^{\frac{3}{2}} - \left[\frac{x^3}{3}\right]_{x=-\sqrt{h}}^{x=\sqrt{h}} = 2h^{\frac{3}{2}} - \left(\frac{2h^{\frac{3}{2}}}{3}\right) = \frac{4}{3}h^{\frac{3}{2}}$$

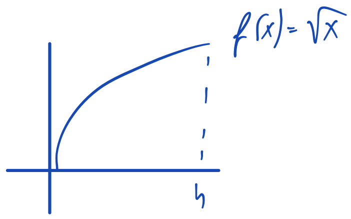
$-h\cdot h^{\frac{1}{2}} = -h^{\frac{3}{2}}$ $\quad h^{\frac{3}{2}} - (-h^{\frac{3}{2}}) = 2h^{\frac{3}{2}}$

$h\cdot h^{\frac{1}{2}} = h^{\frac{3}{2}}$

$$V = \pi \int_a^b f^2(x)\, dx$$

podél x

Chceme rotační objem podél y:

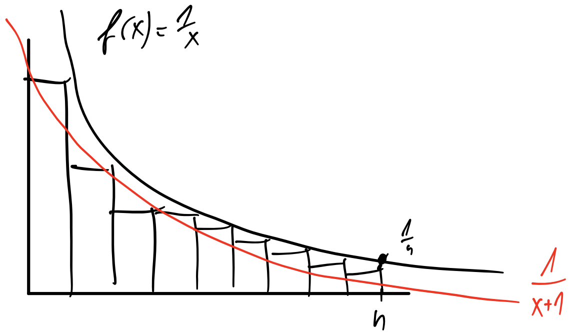


$$V = \pi\cdot\int_0^h \left(\sqrt{x}\right)^2 dx = \pi\left[\frac{x^2}{2}\right]_{x=0}^{x=h} = \pi\frac{h^2}{2}$$

$$H_n = \sum_{k=1}^{n} \frac{1}{k} < 1 + \int_{\cancel{0}\ 1}^{n} \frac{1}{x}\,dx = 1 + \left[\ln x\right]_{x=1}^{n}$$

nejde

$$= 1 + \ln n$$



$$\int_{0}^{n} \frac{1}{x+1}\,dx \qquad \begin{matrix} t = x+1 \\ dt = dx \end{matrix} \qquad = \int_{1}^{n+1} \frac{1}{t}\,dt = \left[\ln t\right]_{t=1}^{t=n+1} = \ln(n+1) > \ln(n)$$

$$\ln(n) < H_n < 1 + \ln(n)$$