Zderivujte:

$$\sqrt{\frac{1-x^2}{1+x^2}}' = \left(\left(\frac{1-x^2}{1+x^2}\right)^{\frac12}\right)' = \frac12\cdot\left(\frac{1-x^2}{1+x^2}\right)^{-\frac12}\cdot\frac{(1-x^2)'\cdot(1+x^2)-(1-x^2)\cdot(1+x^2)'}{(1+x^2)^2} =$$

$$\frac12\cdot\frac{(1+x^2)^{\frac12}}{(1-x^2)^{\frac12}}\cdot\frac{-2x\cdot(1+x^2)-(1-x^2)\cdot(2x)}{(1+x^2)^2} = \frac12\cdot\frac{(1+x^2)^{\frac12}}{(1-x^2)^{\frac12}}\cdot\frac{\overset{-4x}{-2x-2x^3-2x+2x^3}}{(1+x^2)^2} = \frac{(1+x)^{\frac12}}{(1-x)^{\frac12}}\cdot\frac{-2x}{(1+x^2)^2}$$

$$= \dots$$

Spočítejte limity:

AL vyjde $\frac00$, to jde ale L'H

a) $$\lim_{x\to0}\frac{\sin x}{x} \overset{L'H}{=} \lim_{x\to0}\frac{(\sin x)'}{x'} = \lim_{x\to0}\frac{\cos x}{1} \overset{AL}{=} \frac11 = 1$$

1. podmínka VOLSF, cosinus je spojitá f.

b) $$\lim_{x\to\infty} x\cdot\sin\frac1x = \lim_{x\to\infty}\frac{\sin\frac1x}{\frac1x} \xrightarrow{L'H} \frac{\cos\frac1x\cdot\left(\frac1x\right)'}{\left(\frac1x\right)'} = \lim_{x\to\infty}\cos\frac1x = \cos 0 = 1$$

platí zde druhá podmínka VOLSF

$f =$ $\quad g = \frac1x$ $\quad \lim_{x\to\infty} f\left(\frac1x\right)$

c) $$\lim_{x\to1}\frac{x^2+2x-3}{x^2-1} = \lim_{x\to1}\frac{\cancel{(x-1)}\cdot(x+3)}{\cancel{(x-1)}\cdot(x+1)} = \frac{x+3}{x+1} \overset{AL}{=} \frac42 = 2$$

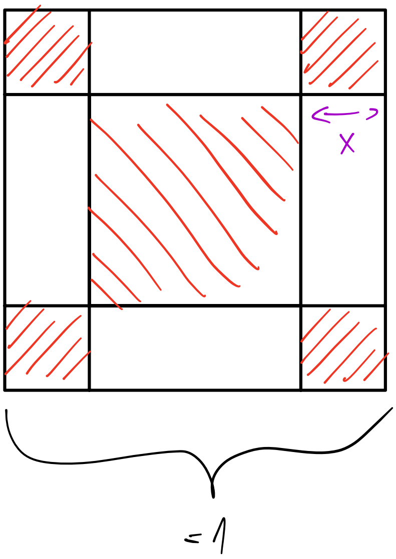
$$L'H = \frac{2x+2}{2x} = \frac{2+2}{2} = 2$$

d) $$\lim_{x\to\infty}\frac{\ln(1+\sqrt{x})}{\ln(1+\sqrt[3]{x})} = \frac{\cancel{\ln(1)}\cdot\ln\sqrt{x}}{\cancel{\ln(1)}\cdot\ln\sqrt[3]{x}} = \frac{\ln x^{\frac12}}{\ln x^{\frac13}} = \frac{\frac12\cancel{\ln x}}{\frac13\cancel{\ln x}} = \frac32$$

$$L'H = \frac{\frac{1}{1+x^{\frac12}}\cdot\frac{1}{2x^{\frac12}}}{\frac{1}{1+x^{\frac13}}\cdot\frac{1}{3x^{\frac23}}} = \frac32\lim_{x\to\infty}\frac{(1+x^{\frac13})\cdot x^{\frac16}}{1+x^{\frac12}} = \frac{x^{\frac16}+x^{\frac12}}{1+x^{\frac12}} \overset{L'H}{=} \frac32\cdot\lim_{x\to\infty}\frac{\frac16\cdot\frac{1}{x^{5/6}}+\frac{1}{2x^{\frac12}}}{\frac{1}{2x^{\frac12}}} =$$

$$(1+x^{\frac12})' = \frac12\cdot x^{-\frac12} = \frac{1}{2x^{\frac12}} \qquad (1+x^{\frac13})' = \frac13\cdot x^{-\frac23} = \frac{1}{3x^{\frac23}}$$

$$= \frac32\lim_{x\to\infty}\underbrace{\left(\frac{1}{3\cdot x^{\frac13}}+1\right)}_{=1} = \frac32$$



Jak moc máme nařezat, ať maximalizujeme objem.

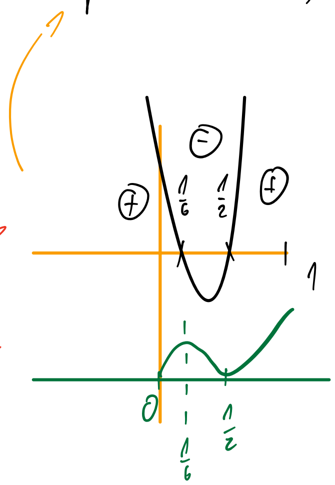
$x \in (0, \frac{1}{2})$

$$V = (1-2x)^2 \cdot x = (1-4x+4x^2)\cdot x = 4x^3 - 4x^2 + x = f$$

$$f' = 12x^2 - 8x + 1$$

Zajímá nás, kde jsou kořeny nulové, protože tam se objevují lokální extr.

$$x = \frac{8 \pm \sqrt{64-48}}{24} = \begin{cases} \frac{1}{2} \\ \frac{1}{6} \end{cases}$$



kde je to kladné, tam je funkce rostoucí, kde je to záporné, tam je funkce klesající

Průběh funkcí:

$f(x): x^3 - 12x + 16$

$D_f: x \in \mathbb{R}$

Průsečíky os:

Pro $x_y = 0 - 12\cdot 0 + 16 \quad [0,16]$

Pro $x_x = (x-2)\cdot\underbrace{(x^2+2x-8)}_{\text{tohle jsme vypočítali}} = (x-2)\cdot(x-2)\cdot(x+4)$ — $[2,0]$ ② Násobnost dva; $[-4,0]$

$$f' = 3x^2 - 12 = 3\cdot(x^2-4)$$

$x'_1 = 2$

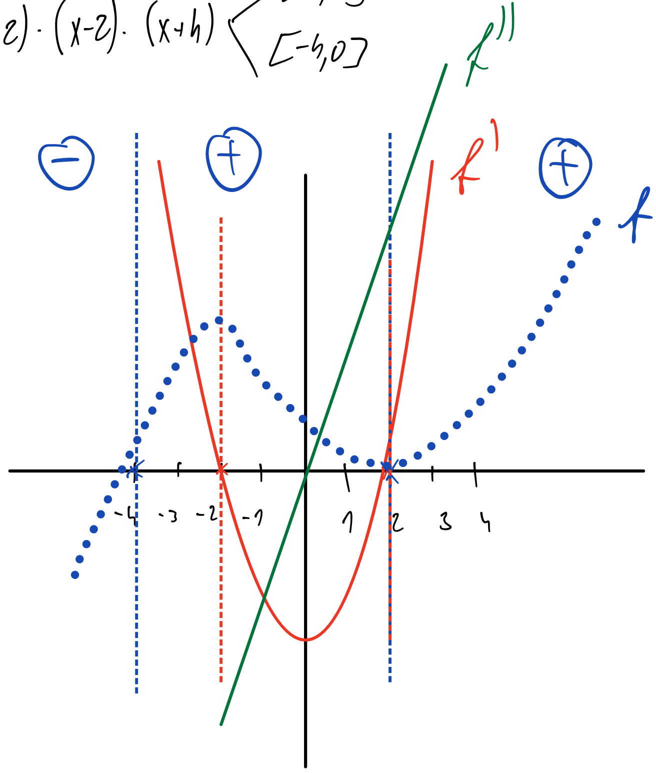
$x'_2 = -2$

$$f'' = 6x$$

$+ \ \cup$ konvexní (lok. max)

$- \ \cap$ konkávní (lok. min)

x je zde kořen f'



$$f(x) = \sqrt[x]{x} = x^{\frac{1}{x}}$$

Df: $x \in \mathbb{R}^+$

Průsečíky:

s y: neexistuje

s x: neexistuje

$$\lim_{x \to 0^+} x^{\frac{1}{x}} = \lim_{x \to 0} e^{x^{-1} \cdot \ln x} = \dots = 0$$

$\nearrow$ VOLSF $\nearrow$ nedopočítali jsme

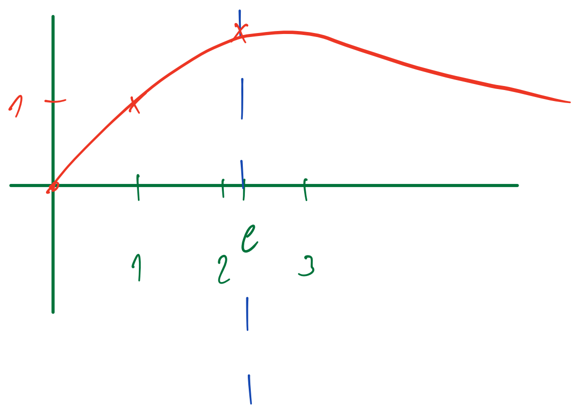
$$\lim_{x \to \infty} x^{\frac{1}{x}} =$$

$$f' = \left(x^{\frac{1}{x}}\right)' = \left(e^{\frac{1}{x}\ln x}\right)' = e^{\frac{1}{x}\ln x} \cdot \left(\left(x^{-1}\right)' \cdot \ln x + \left(x^{-1}\right) \cdot (\ln x)'\right) = e^{\frac{1}{x}\ln x} \cdot \left(-x^{-2} \cdot \ln x + x^{-2}\right) =$$

$$= e^{\frac{1}{x}\ln x} \cdot -x^{-2} \cdot (\ln x - 1)$$

$\nearrow$

$\ln x = 1 \rightarrow x = e$



Mějme $f(x)$, pak v bodě x je f, pokud:

$$f'(x) > 0 \Rightarrow f(x) \nearrow$$

$$f'(x) < 0 \Rightarrow f(x) \searrow$$

$$f''(x) > 0 \Rightarrow f(x) \quad \cap \quad \text{konkávní}$$

$$f''(x) < 0 \Rightarrow f(x) \quad \cup \quad \text{konvexní}$$