

$$(\text{konst})' = 0$$

$$(x)' = 1$$

$$(x^n)' = n \cdot x^{n-1}$$

$$(e^x)' = e^x$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\ln x)' = \frac{1}{x}$$

$$\left(\frac{1}{x}\right)' = \lim_{h \rightarrow 0} \frac{\frac{1}{a+h} - \frac{1}{a}}{h} = \frac{\frac{a - (a+h)}{a \cdot (a+h)}}{h} = \frac{-h}{a^2 + ah} = \frac{-1}{a^2 + ah} = -\frac{1}{a^2}$$

$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$(\sqrt{x})' = (x^{\frac{1}{2}})' = \frac{1}{2\sqrt{x}} = \frac{1}{2} \cdot x^{-\frac{1}{2}}$$

$$\left(\frac{f}{g}\right)' = \left(f \cdot \frac{1}{g}\right)' = f' \cdot \frac{1}{g} + f \cdot \left(\frac{1}{g}\right)' = f' \cdot \frac{1}{g} + f \cdot \frac{-1}{g^2} \cdot g' = \frac{f'g - f \cdot g'}{g^2}$$

Věta:

$$f(x) = y \quad (f^{-1}(y))' = \frac{1}{f'(x)}$$

Derivace inverzní funkce

$$e^x = y$$
$$(\ln y)' = \frac{1}{e^x} = \frac{1}{y}$$

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

Aritmetika:

$$(\alpha f)' = \alpha \cdot f'$$

$$(f \pm g)' = f' \pm g'$$

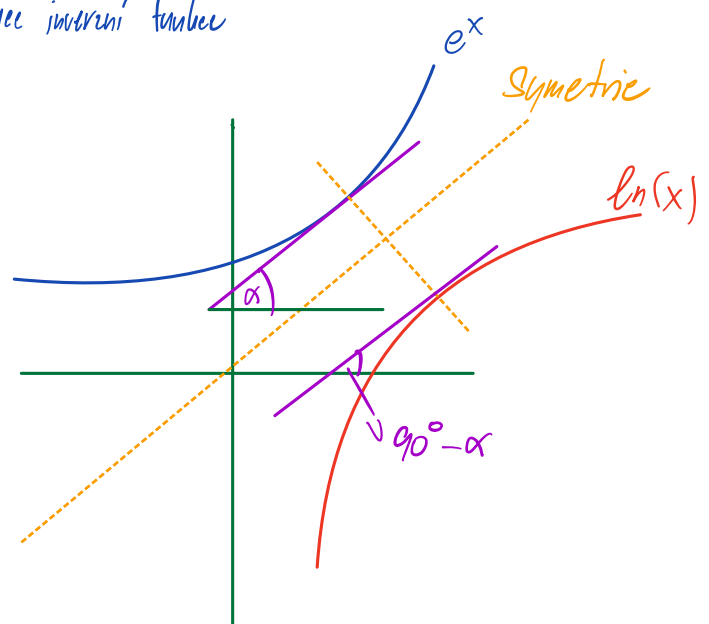
$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - f \cdot g'}{g^2}$$

Stožcová funkce:

$$f(g(a))' = f'(g(a)) \cdot g'(a)$$

protože složná funkce



2. derivujte:

$$\sin^2 x + \cos^2 x = (\sin x \cdot \sin x) + (\cos x \cdot \cos x) = \sin x \cdot \cos x + \cos x \cdot \sin x + \cos x \cdot \sin x - \sin x \cdot \cos x =$$

a)  $(\sin^2 x + \cos^2 x)' = (1)' = 0$

b)  $(x^x)' = (e^{x \cdot \ln x})' = \underbrace{e^{x \ln x}}_{x^x} \cdot (x \cdot \ln x)' = x^x \cdot (\ln x + x \cdot \frac{1}{x}) = x^x \cdot (1 + \ln x)$

! c)  $(x^{\ln x})' = x^{\ln x} \cdot \frac{2 \ln x}{x}$

! d)  $((\ln x)^x)' = (\ln x)^x \cdot (\ln \ln x) + (\ln x)^{x-1}$

e)  $(\arctan \frac{1}{x})' \rightarrow (\tan x)' = (\frac{\sin x}{\cos x})' = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = 1 + \tan^2 x$

$(\frac{\sin x}{\cos x})' = (\tan x)'$

$\rightarrow (\arctan x)' \rightarrow \tan x = y \Rightarrow (\arctan y)' = \frac{1}{1 + \tan^2 x} = \frac{1}{1 + y^2}$

$\rightarrow (\arctan \frac{1}{x})' = \frac{1}{1 + (\frac{1}{x})^2} \cdot (-\frac{1}{x^2}) = \frac{-1}{\frac{x^2+1}{x^2} \cdot x^2} = -\frac{1}{1+x^2}$

2. derivujte:

$f(g(x))' = f'(g(x)) \cdot g'$

a)  $(\cos^3(2x))' = 3 \cdot \cos^2 2x \cdot (-\sin 2x) \cdot 2 = -6 \cdot \sin 2x \cdot \cos^2 2x$

b)  $(\sin \sqrt{x-1})' = \cos \sqrt{x-1} \cdot \frac{1}{2\sqrt{x-1}} \cdot 1$

c)  $(\sqrt{\frac{1-x^2}{1+x^2}})'$

d)  $(x^2 \cdot 2^x)' = 2x \cdot 2^x + x^2 \cdot (2^x)' = 2x \cdot 2^x + x^2 \cdot 2^x \cdot \ln 2$

$(c^x)' = (e^{x \ln c})' = \underbrace{e^{x \ln c}}_{c^x} \cdot \ln c$

e)  $(x^{x^x})'$

$e^{x \ln x} = a = x^x$

$\ln e^{x \ln x} = \ln_e(a)$

$x \ln x \cdot \cancel{\ln e} = \ln_e(a)$

$\ln(x^x) = \ln_e(a)$   
 $x^x = a$

$e^{a \ln b} = x = b^a$

$\ln_e a \ln b = \ln x$

$a \ln b \cdot \cancel{\ln e} = \ln x$

$\ln b^a = \ln x$

$b^a = x$