1) $\lim\limits_{n\to\infty} \ln\left(1+\frac{1}{\sqrt{n}}\right) = ?$ $\to$ $\frac{\sqrt{n}+1}{\sqrt{n}} \cdot \frac{\sqrt{n}}{\sqrt{n}} = \frac{n+\sqrt{n}}{n}$

- jelikož nemůžeme na posloupnosti použít aritmetiku limit, převedu to na funkci.

$\frac{n}{n} \leq \frac{n+\sqrt{n}}{n} \leq$

$\lim\limits_{x\to\infty} \ln\left(1+\frac{1}{\sqrt{x}}\right)$

$f() = \ln()$

$g(x) = \left(1+\frac{1}{\sqrt{x}}\right) \to \lim\limits_{x\to\infty} \left(1+\frac{1}{\sqrt{x}}\right) = 1+0 = 1$

a) $\lim\limits_{x\to\infty} \left(1+\frac{1}{\sqrt{x}}\right) = A$

$\lim\limits_{x\to\infty} g(x) = 1 = A$

b) $\lim\limits_{x\to\infty} (A) = B$

$\to$ Tato funkce je spojitá v celém intervalu

$\lim\limits_{x\to 1} f(A) = 0$

$f(g(x)) = ?$

$f(g(x)) = 0$

$P(a,\varepsilon)$

$\exists \varepsilon>0\ \forall x\in\left(\frac{1}{\varepsilon}, +\infty\right) : g(x) \neq A$

Takto se používá limita složení funkce na limitu posloupnosti

$\lim\limits_{x\to 0} \frac{\sin x}{x} = 1$

$\lim\limits_{x\to 0} \frac{e^{x}-1}{x} = 1$

$\lim\limits_{x\to 0} \frac{\ln(1+x)}{x} = 1$

a) $\lim\limits_{x\to 0} \frac{\ln(1+\sin x)}{x} = 1$

a) $\frac{\ln(1+\sin x)}{x} = \frac{\ln(1+\sin x)}{\sin x} \cdot \frac{\sin x}{x}$, $\frac{\ln(1+\sin x)}{\sin x} \to 1$, $\frac{\sin x}{x} \to 1$

limita složení funkce

b) $\lim\limits_{x\to 0} \sqrt[x]{x+1}$

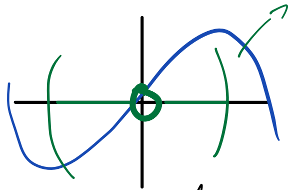
$g(x) = \sin x \to \lim\limits_{x\to 0} \sin x = 0$??

$f(A)\ \frac{\ln(1+A)}{A} \to \lim\limits_{x\to 0} \frac{\ln(1+x)}{x} = 1$

$f(x)^{g(x)} = e^{\ln f(x)^{g(x)}} = e^{g(x)\cdot \ln f(x)}$

$\exists \varepsilon>0\ \forall x\in P(0,\varepsilon) : \sin x \neq 0$

- limita složení funkce



- prstencové, tedy bez nuly

b) $\lim\limits_{x\to 0} (x+1)^{1/x} \to (x+1)^{1/x} = e^{\ln(x+1)^{1/x}} = e^{\frac{1}{x}\ln(x+1)}$

$\lim\limits_{x\to 0} \frac{1}{x}\ln(x+1) = 1$

$\to$ složení funkce

$e^{1} \Rightarrow \lim\limits_{x\to 0} (x+1)^{1/x} = e$

c) $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2} \rightsquigarrow \dfrac{1-\cos x}{x^2}\cdot\dfrac{1+\cos x}{1+\cos x} = \dfrac{1-\cos^2 x}{x^2\cdot(1+\cos x)} = \dfrac{\sin^2 x}{x^2(1+\cos x)} = \dfrac{\sin^2 x}{x^2}\cdot\dfrac{1}{1+\cos x} \to \dfrac{1}{2} \Rightarrow \lim\limits_{x\to 0}\dfrac{1-\cos x}{x^2} = \dfrac{1}{2}$

$\left(\dfrac{\sin x}{x}\right)^2 = 1^2 = 1$

d) $\lim\limits_{x\to 0} \dfrac{\operatorname{tg} x}{x} \rightsquigarrow \dfrac{\frac{\sin x}{\cos x}}{x} = \dfrac{\sin x}{\cos x}\cdot\dfrac{1}{x} = \dfrac{\sin x}{x\cdot\cos x} = \underbrace{\dfrac{\sin x}{x}}_{\to 1}\cdot\underbrace{\dfrac{1}{\cos x}}_{=1} \Rightarrow \lim\limits_{x\to 0}\dfrac{\operatorname{tg} x}{x} = 1$

$\dfrac{1}{\cos x}$ můžu přímo dosadit, jelikož je to funkce, která je celá spojitá, tedy je definovaná i v té limitě a bude se tomu rovnat

e) $\lim\limits_{x\to 0} \dfrac{1}{x^n}$, $n\in\mathbb{N}$ $= \begin{cases} n \text{ liché: neexistuje (stejné jako níže)} \\ n \text{ sudé: } +\infty \text{ (všechno je kladné, takže je to jako limita zprava)} \end{cases}$

zprava $\lim\limits_{x\to 0^+} \dfrac{1}{x} = +\infty$

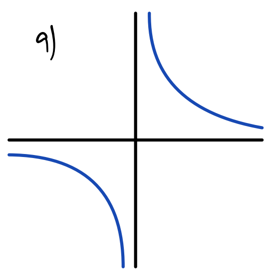
zleva $\lim\limits_{x\to 0^-} \dfrac{1}{x} = -\infty$

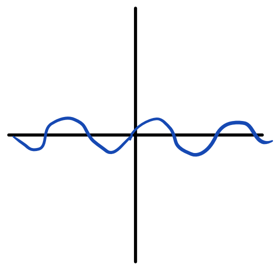
$+\infty \neq -\infty \Rightarrow \lim\limits_{x\to 0}\dfrac{1}{x} =$ neexistuje

Určete def. obor, spojitost, rozmyslete si podobu grafu

a) $\dfrac{1}{x}$ $\quad D(f): \mathbb{R}\setminus\{0\},\ S(f) = \mathbb{R}\setminus\{0\}$

b) $\sin x$ $\quad D(f): \mathbb{R},\ S(f) = \mathbb{R}$

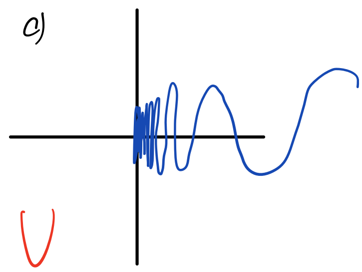




c) $\sin\frac{1}{x}$ $\quad D(f): \mathbb{R}\setminus\{0\},\ S(f) = \mathbb{R}\setminus\{0\}$ $\quad \lim\limits_{x\to 0}\sin\frac{1}{x}$ neexistuje

↳ v nule nelze spojitě dodefinovat

d) $\dfrac{1}{\sin x}$ $\quad D(f) = S(f): \mathbb{R}\setminus\{x\in\mathbb{R}: \sin x = 0\} = \{k\pi \mid k\in\mathbb{Z}\}$

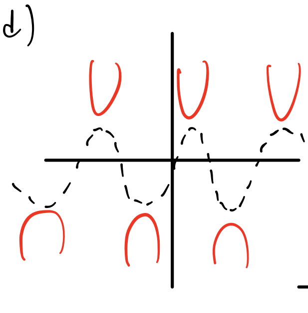


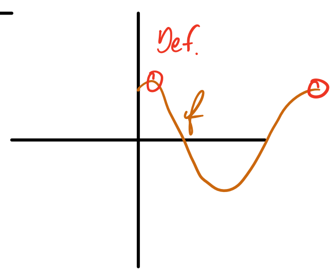
e) $\dfrac{\cos^2 x}{1-\sin x}$ $\quad D(f) = \mathbb{R}\setminus\{x\in\mathbb{R}: \sin x = 1\} = \{\frac{\pi}{2}+2k\pi \mid k\in\mathbb{Z}\}$

$S(f) = \mathbb{R}\setminus\{\frac{\pi}{2}+2k\pi \mid k\in\mathbb{Z}\}$

podíl spojitých je spojitý

podíl dvou spojitých je zase spojitý





$\lim\limits_{x\to\frac{\pi}{2}} \dfrac{\cos^2 x}{1-\sin x} = \lim \dfrac{1-\sin^2 x}{1-\sin x} = \lim 1+\sin x = \underline{\underline{2}}$