

<u>Funkee</u>	<u>Derivace</u>	<u>Primitivní funkce</u>
$f(x)$	$f'(x)$	$\int f(x)$
$x^n$	$n \cdot x^{n-1} \quad (n \neq 0)$	$\frac{1}{n+1} x^{n+1} \quad (n \neq -1)$
$\frac{1}{x} \quad (x \neq 0)$	$-\frac{1}{x^2}$	$\ln x $
$e^x$	$e^x$	$e^x$
$\sin x$	$\cos x$	$-\cos x$
$\cos x$	$-\sin x$	$\sin x$
$\ln x$	$\frac{1}{x}$	$x \cdot \ln x - x$
$\operatorname{tg} x$	$\frac{1}{\cos^2 x}$	
$\operatorname{arctg} x$	$\frac{1}{1+x^2}$	

interval  
nemí robitky  $\rightarrow$

### Spočítejte integrál:

a)  $\int (x^2 + 2x) dx$

b)  $\int (x+2)^2 dx$

c)  $\int e^x - e^{-x} dx$

d)  $\int \sin x - \cos x dx$

e)  $\int \frac{1+x}{\sqrt{x}} dx$

f)  $\int \tan^2 x dx$

g)  $\int x \cdot e^{-x^2} dx$

a)  $\frac{x^3}{3} + x^2 + c$

b)  $\rightarrow -x^2 + hx + h \rightarrow \frac{x^3}{3} + 2x^2 + hx + c$

$t = x+2$   
 $dt = dx = \int t^2 dt = \frac{t^3}{3} = \frac{(x+2)^3}{3} + c$

a)  $\int e^x dx + \int e^{-x} dx = e^x + e^{-x} + c$

d)  $(\int \sin x - \int \cos x) dx = -\cos x - \sin x + c$

e)  $\int \frac{1+x}{x^{\frac{1}{2}}} dx = \int (x^{-\frac{1}{2}} + x^{\frac{1}{2}}) dx = \int x^{-\frac{1}{2}} dx + \int x^{\frac{1}{2}} dx = 2 \cdot x^{\frac{1}{2}} + \frac{2}{3} x^{\frac{3}{2}} + c$

f)  $\int \frac{\sin^2 x}{\cos^2 x} dx = \int \frac{1-\cos^2 x}{\cos^2 x} dx = \int \frac{1}{\cos^2 x} - 1 dx = \operatorname{tg} x - x + c$

g)  $-\frac{1}{2} \int e^{-x^2} \cdot (-2x) dx = -\frac{1}{2} \cdot e^{-x^2} + c$

$t = -x^2$   
 $dt = -2x dx$   
 $= \int e^t \cdot \frac{dt}{-2} = -\frac{1}{2} \int e^t dt = -\frac{1}{2} e^t + c = -\frac{1}{2} e^{-x^2} + c$

$\int (x+2)^2 dx$   
 $\int (x+2) \cdot 1 dx$   
 $\int 1 \cdot 2 dx$

$f(g(x)) = (f'(g(x)) \cdot g'(x)) dx$

## Spočítejte integrály:

a)  $\int \tan x \, dx$

b)  $\int \sin^2 x \, dx$

c)  $\int \cos^2 x \, dx$

$$\cos x = t$$

a)  $\int \frac{\sin x}{\cos x} \, dx = - \int \frac{1}{\cos x} \cdot (-\sin x) \cdot dx = - \ln|\cos x| + c$



$$t = \cos x \\ dt = -\sin x \, dx$$



$$- \int \frac{-\sin x}{\cos x} \, dx = - \int \frac{1}{t} \, dt = - \ln|t| = - \ln|\cos x| + c$$

b)  $\left. \begin{aligned} \sin^2 x + \cos^2 x &= 1 \\ \cos^2 x - \sin^2 x &= \cos 2x \end{aligned} \right\} \sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$$\frac{1}{2} \int 1 - \cos 2x \, dx = \frac{1}{2} (x - \sin x \cos x) + c$$

$$\int \cos 2x \, dx = \frac{1}{2} \sin 2x = \sin x \cos x \\ t = 2x \\ dt = 2 \, dx$$

$$\underline{f(x) \cdot g(x) = \int f(x) \cdot g'(x) \, dx + \int f'(x) \cdot g(x) \, dx}$$

## Derivace plus partes:

$$\int f(x) \cdot g'(x) \, dx = f(x) \cdot g(x) - \int f'(x) \cdot g(x) \, dx$$

## Spočítejte integrály:

a)  $\int x \cdot e^x \, dx$

b)  $\int x \cdot \sin x \, dx$

a)  $\int x \cdot e^x \, dx = x \cdot e^x - \int 1 \cdot e^x \, dx = x \cdot e^x - e^x + c$

b)  $\int x \cdot \sin x \, dx = -x \cdot \cos x - \int 1 \cdot (-\cos x) \, dx = -x \cdot \cos x + \sin x + c$

## Opačkováni: Spočítejte integrály:

$$\int x \cdot e^{-x^2} \, dx =$$

$$t = -x^2 \\ dt = -2x \, dx$$

$$\int \ln x \, dx = \int \ln x \cdot 1 \, dx$$

$$f = \ln x \quad f' = \frac{1}{x} \\ g = x \quad g' = 1$$