Determinanty

Určete determinant:

a) $$\begin{pmatrix} 0 & b & 1 & 0 \\ a & 0 & 1 & 1 \\ 1 & 1 & 0 & d \\ 0 & 1 & c & 0 \end{pmatrix} = -a \cdot \begin{vmatrix} b & 1 & 0 \\ 1 & 0 & d \\ 1 & c & 0 \end{vmatrix} + \begin{vmatrix} b & 1 & 0 \\ 0 & 1 & 1 \\ 1 & c & 0 \end{vmatrix} = -a\cdot(d - c\cdot b\cdot d) + 1 + cb = -ad + abcd + cb + 1$$

Cramerovo pravidlo:

Vyřešte soustavu: $Ax = b$

$$A = \begin{pmatrix} 1 & 2 & -2 & 2 \\ 2 & 3 & 1 & 1 \\ -1 & 2 & 3 & -1 \\ 1 & -1 & 1 & -1 \end{pmatrix}, \quad b = \begin{pmatrix} -2 \\ 2 \\ -1 \\ 4 \end{pmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & 2 & -2 & 2 \\ 0 & -1 & 5 & -3 \\ 0 & 4 & 1 & 1 \\ 0 & -3 & 3 & -3 \end{vmatrix} = \begin{vmatrix} 1 & 2 & -2 & 2 \\ 0 & -1 & 5 & 3 \\ 0 & 0 & 21 & -11 \\ 0 & 0 & -12 & 6 \end{vmatrix} = \begin{vmatrix} 1 & 2 & -2 & 2 \\ 0 & -1 & 5 & 3 \\ 0 & 0 & 21 & -11 \\ 0 & 0 & 0 & -2/7 \end{vmatrix} = 6$$

$$x_1 = \begin{vmatrix} -2 & 2 & -2 & 2 \\ 2 & 3 & 1 & 1 \\ -1 & 2 & 3 & -1 \\ 4 & -1 & 1 & -1 \end{vmatrix} = \begin{vmatrix} -2 & 2 & -2 & 2 \\ 0 & 5 & -1 & 3 \\ 0 & 1 & 4 & -2 \\ 0 & 3 & -3 & 3 \end{vmatrix} = \begin{vmatrix} -2 & 2 & -2 & 2 \\ 0 & 5 & -1 & 3 \\ 0 & 0 & 21/5 & -13/5 \\ 0 & 0 & -12/5 & 6/5 \end{vmatrix} = \begin{vmatrix} -2 & 2 & -2 & 2 \\ 0 & 5 & -1 & 3 \\ 0 & 0 & 21/5 & -13/5 \\ 0 & 0 & 0 & -2/7 \end{vmatrix} = -10 \cdot \frac{\cancel{21}^{3}}{5} \cdot \left(-\frac{2}{\cancel{7}_{1}}\right) = \frac{6}{5}\cdot 10 = 12$$

$x_1 = 12/6 = 2$

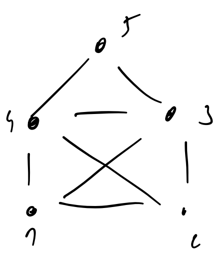
Atd, Atd...

Najděte A^{-1} pomocí Adj. Platí, že $A^{-1} = \dfrac{Adj(A)}{\det(A)}$

$$A = \begin{pmatrix} 1 & 0 & 1 \\ -2 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \qquad \det(A) = 1 - 2 = -1$$

$$Adj(A) = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & -2 \\ -2 & -1 & 1 \end{pmatrix} \qquad A^{-1} = \begin{pmatrix} -1 & -1 & 1 \\ -2 & -1 & 2 \\ 2 & 1 & -1 \end{pmatrix}$$

Počet koster grafu:



$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{pmatrix} 3 & -1 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 & 0 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & -1 & 4 & -1 \\ 0 & 0 & -1 & -1 & 2 \end{pmatrix} \end{array}$$

→ Spočít determinant!

Převeďte matici do tvaru $A = RJR^{-1}$

t^2-4t+4

$3-3t-t+t^2$

$$A = \begin{pmatrix}1&1&1\\0&1&0\\-1&0&3\end{pmatrix} \qquad p_A(t) = \begin{vmatrix}1-t&1&1\\0&1-t&0\\-1&0&3-t\end{vmatrix} = (1-t)^2\cdot(3-t) + (1-t) = (1-t)\cdot\big((1-t)\cdot(3-t)+1\big) = (1-t)\cdot(t-2)^2$$

$$\lambda = 1 \begin{pmatrix}0&1&1\\0&0&0\\-1&0&2\end{pmatrix} \sim \begin{pmatrix}1&0&-2\\0&1&1\\0&0&0\end{pmatrix}$$

$\dim(\mathrm{Ker}(A-1\cdot I_n)) = 1$ → geometrická násobnost

→ $x_3 = t$, $x_2 = -t$, $x_1 = 2t$ $\quad x = c\cdot(2,-1,1)^T$

$$\lambda_2 = 2 \begin{pmatrix}-1&1&1\\0&-1&0\\-1&0&1\end{pmatrix} \sim \begin{pmatrix}1&-1&-1\\0&1&0\\0&-1&0\end{pmatrix} \sim \begin{pmatrix}1&0&-1\\0&1&0\\0&0&0\end{pmatrix}$$

$\dim(\mathrm{Ker}(A-2\cdot I_n)) = 1$ → geometrická násobnost.

→ $x_3 = t$, $x_2 = 0$, $x_1 = t$ $\quad x = c\cdot(1,0,1)^T$

$$J = \begin{pmatrix}2&1&0\\0&2&0\\0&0&1\end{pmatrix}$$

$AR = RJ$

$$\begin{array}{c|ccc} & R & & \\ & 2 & 1 & ? \\ & -1 & 0 & \\ & 1 & 1 & \\ \hline A & & & \\ 1\ 1\ 1 & | & | & | \\ 0\ 1\ 0 & Au_1 & Au_2 & Au_3 \\ -1\ 0\ 3 & | & | & | \end{array} \qquad \begin{array}{c|ccc} & J & & \\ & 1 & 0 & 0 \\ & 0 & 2 & 1 \\ & 0 & 0 & 2 \\ \hline R & & & \\ 2\ 1\ ? & | & | & | \\ -1\ 0 & u_1 & 2u_2 & u_2+2u_3 \\ 1\ 1 & | & | & | \end{array}$$

$Au_1 = u_1 \qquad u_1 = (2,-1,1)^T$

$Au_2 = 2u_2 \qquad u_2 = (2,0,2)^T$

$Au_3 = u_2 + 2u_3$

$Au_3 - 2u_3 = u_2$

$(A-2I)\cdot u_3 = u_2 \qquad Ax = b$

$$R = \begin{pmatrix}2&1&-2\\-1&0&0\\0&1&0\end{pmatrix}$$

$$\left(\begin{array}{ccc|c}-1&1&1&2\\0&-1&0&0\\-1&0&1&2\end{array}\right) \sim \left(\begin{array}{ccc|c}1&0&-1&-2\\0&1&0&0\\0&0&0&0\end{array}\right) \qquad \begin{aligned}x_3&=t\\x_2&=0\\x_1&=2-t\end{aligned}$$

$u_3 = (2-t, 0, t)$

Určete úhel:

$$\langle u|w\rangle = \|u\|\cdot\|w\|\cdot\cos\alpha \;\to\; \cos\alpha = \frac{\langle u|w\rangle}{\|u\|\cdot\|w\|} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} \Rightarrow \alpha = \frac{3}{4}\pi$$

$x^T = (0,0,1)$, $y^T = (1,0,-1)$ $\qquad \langle x|y\rangle = -1$

$\|u\| = 1$

$\|y\| = \sqrt{2}$