Domácí úlohy — sada č. 6

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Potřebný čas:

1. Rozložte následující matici **A** do Jordanova normálního tvaru $\mathbf{A} = \mathbf{R}\mathbf{J}\mathbf{R}^{-1}$

$$\mathbf{A} = \begin{pmatrix} 2 & 6 & -7 \\ -3 & -7 & 8 \\ -1 & -2 & 2 \end{pmatrix}$$

a proveďte zkoušku.

$$p_A(t) = \begin{vmatrix} 2-t & 6 & -7 \\ -3 & -7-t & 8 \\ -1 & -2 & 2-t \end{vmatrix} = (2-t)^2\cdot(-7-t) - 48 - 42 + 49 + 7t + 32 - 16t + 36 - 18t =$$

$$= (2-t)^2\cdot(-7-t) + 27 - 27t = -28 + 28t - 7t^2 - 4t + 4t^2 - t^3 + 27 - 27t$$

$$= -t^3 - 3t^2 - 3t - 1 = 0 = -(t+1)^3$$

$\underline{\lambda_1 = -1,\ \text{alg. nás.} = 3}$

$$\begin{pmatrix} 3 & 6 & -7 \\ -3 & -6 & 8 \\ -1 & -2 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & -3 \\ 0 & 0 & -1 \\ 0 & 0 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad \left.\begin{matrix} x_3 = 0 \\ x_2 = t \\ x_1 = -2t \end{matrix}\right\} \ v_1 = c\cdot(-2,1,0)^T$$

$AR = RJ$

$A = RJR^{-1}$

$$\begin{array}{c|ccc} & u_1 & u_2 & u_3 \\ R & -2 & \cdot & \cdot \\ & 1 & \cdot & \cdot \\ & 0 & \cdot & \cdot \\ \hline A & & & \\ 2\ \ 6\ \ -7 & Au_1 & Au_2 & Au_3 \\ -3\ \ -7\ \ 8 & & & \\ -1\ \ -2\ \ 2 & & & \end{array} \qquad \begin{array}{c|ccc} & -1 & 1 & 0 \\ & 0 & -1 & 1 \\ & 0 & 0 & -1 \\ \hline R\ \ u_1\ u_2\ u_3 & & & \\ -2\ \cdot\ \cdot & -u_1 & u_1-u_2 & u_2-u_3 \\ 1\ \cdot\ \cdot & & & \\ 0\ \cdot\ \cdot & & & \end{array}$$

a) $$\left(\begin{array}{ccc|c} 3 & 6 & -7 & -2 \\ -3 & -6 & 8 & 1 \\ 1 & -2 & 3 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 0 & 1 & -1 \\ 3 & 6 & -7 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 12 & -16 & -2 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & -2 & 0 & 3 \\ 0 & 12 & 0 & -18 \\ 0 & 0 & 1 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 0 & 3 \\ 0 & 2 & 0 & -3 \\ 0 & 0 & 1 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\frac{3}{2} \\ 0 & 0 & 1 & -1 \end{array}\right)$$

b) $$\left(\begin{array}{ccc|c} 3 & 6 & -7 & 0 \\ -3 & -6 & 8 & -\frac{3}{2} \\ 1 & -2 & 3 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ 0 & 0 & 1 & -\frac{3}{2} \\ 3 & 6 & -7 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ 0 & 12 & -16 & 3 \\ 0 & 0 & 1 & -\frac{3}{2} \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & -2 & 0 & \frac{7}{2} \\ 0 & 12 & 0 & -21 \\ 0 & 0 & 1 & -\frac{3}{2} \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 0 & \frac{7}{2} \\ 0 & 2 & 0 & -\frac{21}{6} \\ 0 & 0 & 1 & -\frac{3}{2} \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\frac{7}{4} \\ 0 & 0 & 1 & -\frac{3}{2} \end{array}\right)$$

$Au_1 = -u_1$

$Au_2 = u_1 - u_2 \quad Au_2 + u_2 = u_1 \quad (A+I)u_2 = u_1$ a) $(0, -\frac{3}{2}, -1)^T$

$Au_3 = u_2 - u_3 \quad Au_3 + u_3 = u_2 \quad (A+I)u_3 = u_2$ b) $(0, -\frac{7}{4}, -\frac{3}{2})^T$

$$\left(\begin{array}{ccc|ccc} -2 & 0 & 0 & 1 & 0 & 0 \\ 1 & -\frac{3}{2} & -\frac{7}{4} & 0 & 1 & 0 \\ 0 & -1 & -\frac{3}{2} & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & -\frac{3}{2} & -\frac{7}{4} & 0 & 1 & 0 \\ 0 & -3 & -\frac{7}{2} & 1 & 2 & 0 \\ 0 & -1 & -\frac{3}{2} & 0 & 0 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & -\frac{3}{2} & -\frac{7}{4} & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 2 & -3 \\ 0 & 1 & \frac{3}{2} & 0 & 0 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & -\frac{3}{2} & -\frac{7}{4} & 0 & 1 & 0 \\ 0 & 1 & 0 & -\frac{3}{2} & -3 & \frac{7}{2} \\ 0 & 0 & 1 & 1 & 2 & -3 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & -\frac{3}{2} & -3 & \frac{7}{2} \\ 0 & 0 & 1 & 1 & 2 & -3 \end{array}\right)$$

$$R = \begin{pmatrix} -2 & 0 & 0 \\ 1 & -\frac{3}{2} & -\frac{7}{4} \\ 0 & -1 & -\frac{3}{2} \end{pmatrix} \quad R^{-1} = \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ -\frac{3}{2} & -3 & \frac{7}{2} \\ 1 & 2 & -3 \end{pmatrix}$$

Zk:

$A = RJR^{-1} \qquad RJ = \begin{pmatrix} 2 & -2 & 0 \\ -1 & \frac{5}{2} & \frac{1}{4} \\ 0 & 1 & \frac{1}{2} \end{pmatrix} \qquad RJ\cdot R^{-1} = \begin{pmatrix} 2 & 6 & -7 \\ -3 & -7 & 8 \\ -1 & -2 & 2 \end{pmatrix}$

pokračování na 2. straně

2. Zjistěte pro jaká komplexní čísla z je následující matice unitární:

$z = a + bi$

$$A = \begin{pmatrix} z & \overline{z} \\ iz & -i\overline{z} \end{pmatrix} \quad A^H = \begin{pmatrix} \overline{z} & -i\overline{z} \\ z & iz \end{pmatrix}$$

Unitární matice znamená: $A^H = A^{-1}$

Unitární matice splňuje: $\underline{\underline{A^H \cdot A = I}}$

$$\begin{pmatrix} z & \overline{z} \\ iz & -i\overline{z} \end{pmatrix} \cdot \begin{pmatrix} \overline{z} & -i\overline{z} \\ z & iz \end{pmatrix} = \begin{pmatrix} z\overline{z} + z\overline{z} & iz\overline{z} - iz\overline{z} \\ iz\overline{z} - iz\overline{z} & -i^2 z\overline{z} + -i^2 z\overline{z} \end{pmatrix} = \begin{pmatrix} 2z\overline{z} & 0 \\ 0 & 2z\overline{z} \end{pmatrix}$$

– Rovnou nám vyšly nuly mimo diagonálu -> tím pádem stačí splnit podmínku, že $a_{ii} = 1$

$$2z\overline{z} = 1 \Rightarrow z\overline{z} = (a+bi)\cdot(a-bi) = a^2 + b^2 = \frac{1}{2}$$

$z = a + bi$

tzn. že to platí pro všechna $z = a + bi$, kde $a^2 + b^2 = \frac{1}{2}$.