$$A \begin{pmatrix} 2 & 1 & 0 & 0 \\ 4 & 2 & 0 & 0 \\ 4 & 3 & 0 & 1 \\ 0 & 1 & 4 & 3 \end{pmatrix}, \quad P_A(t) = \begin{vmatrix} 2-t & 1 & 0 & 0 \\ 4 & 2-t & 0 & 0 \\ 4 & 3 & -t & 1 \\ 0 & 1 & 4 & 3-t \end{vmatrix} = -t \begin{vmatrix} 2-t & 1 & 0 \\ 4 & 2-t & 0 \\ 0 & 1 & 3-t \end{vmatrix}$$

$$-4 \cdot \begin{vmatrix} 2-t & 1 & 0 \\ 4 & 2-t & 0 \\ 4 & 3 & 1 \end{vmatrix} = -t\left((2-t)^2 \cdot (3-t) - 4 \cdot (3-t)\right) - 4 \cdot \left((2-t)^2 - 4\right) =$$

$$-t \cdot (2-t)^2 \cdot (3-t) + 12t - 4t^2 - 4(2-t)^2 + 16 =$$

$$t^4 - 4t^3 + 4t^2 - 3t^3 + 12t^2 - 12t + 12t - 4t^2 - 16 + 16t - 4t^2 + 16 =$$

$$t^4 - 7t^3 + 8t^2 + 16t = t \cdot (t^3 + 5t^2 + 8t + 16) = 0 \qquad t \cdot (t+4)^3 = 0$$

$\lambda_1 = \underline{0}, \quad \lambda_2 = (-4) \to \underline{4}$

$$A_0 \begin{pmatrix} 2 & 1 & 0 & 0 \\ 4 & 2 & 0 & 0 \\ 4 & 3 & 0 & 1 \\ 0 & 1 & 4 & 3 \end{pmatrix} \sim \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 1 \\ 0 & 1 & 4 & 3 \end{pmatrix} \sim \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 3 & 3 \end{pmatrix}$$

$x_4 = t = t$
$x_3 = -\frac{t}{2} = 2t$
$x_2 = -t = 4t$
$x_1 = \frac{t}{2} = t$

$v_1 = c \cdot (1, 4, 2, 1)^T$

$$A_4 \begin{pmatrix} -2 & 1 & 0 & 0 \\ 4 & -2 & 0 & 0 \\ 4 & 3 & -4 & 1 \\ 0 & 1 & 4 & -1 \end{pmatrix} \sim \begin{pmatrix} 3 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 4 & 4 \end{pmatrix} \sim \begin{pmatrix} 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$x_4 = t = 1$
$x_3 = -t = 4$
$x_2 = 0 = 0$
$x_1 = 0 = 0$

$v_2 = c \cdot (0, 0, 4, 1)^T$

Není diagonalizovatelná, jelikož λ_2 má alg. násobnost 3, ale jenom geometrickou pouze 1.

V $\mathbb{Z}_{11}$ A^{1000} ?

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{pmatrix} \quad P_A t = \begin{vmatrix} 1-t & 2 & 3 \\ 3 & 4-t & 5 \\ 5 & 6 & 7-t \end{vmatrix} = -t^3 + 12t^2 - 35t + 28 + 50 + 54 - 60 + 15t - 30 + 30t - 42 + 6t$$

$$= 10t^3 + t^2 + t = 0$$

$$t \cdot (10t^2 + t + 1) = 0 \quad \rightarrow$$ Tři vlast. čísla pro matici 3×3

$$\frac{10 \pm \sqrt{1 - 40}}{20} \quad \frac{10 \pm \sqrt{5}}{20} = \frac{10 \pm 4}{4} = \begin{cases} \frac{3}{9} = 4 \\ \frac{6}{7} = 8 \end{cases}$$

$\lambda_1 = 0$

$\lambda_2 = 4$

$\lambda_3 = 8$

$$A_0 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & 9 & 7 \\ 0 & 7 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & 7 & 3 \\ 0 & 1 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$v_1 = c \cdot (1, 9, 1)^T$

$$A_4 = \begin{pmatrix} 8 & 2 & 3 \\ 3 & 0 & 5 \\ 5 & 6 & 3 \end{pmatrix} \sim \begin{pmatrix} 3 & 0 & 5 \\ 3 & 7 & 0 \\ 5 & 6 & 3 \end{pmatrix} \sim \begin{pmatrix} 3 & 0 & 5 \\ 0 & 7 & 6 \\ 2 & 6 & 9 \end{pmatrix} \sim \begin{pmatrix} 1 & 5 & 7 \\ 0 & 7 & 6 \\ 0 & 7 & 6 \end{pmatrix} \sim \begin{pmatrix} 1 & 5 & 7 \\ 0 & 7 & 6 \\ 0 & 0 & 0 \end{pmatrix}$$

$x_3 = t$

$7x_2 = -6t \quad x_2 = -\frac{6}{7}t = -5t = 7t$

$x_1 = -35t - 7t = 2t$

$v_2 = c \cdot (2, 7, 1)^T$

$$A_8 = \begin{pmatrix} 4 & 2 & 3 \\ 3 & 7 & 5 \\ 5 & 6 & 10 \end{pmatrix} \sim \begin{pmatrix} 1 & 6 & 9 \\ 3 & 7 & 5 \\ 6 & 6 & 10 \end{pmatrix} \sim \begin{pmatrix} 1 & 6 & 9 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 6 & 9 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$x_3 = t$

$x_2 = 10t$

$x_1 = -69t = 8t$

$v_3 = c \cdot (8, 10, 1)^T$

$$R = \begin{pmatrix} 1 & 2 & 8 \\ 9 & 7 & 10 \\ 1 & 1 & 1 \end{pmatrix} \rightsquigarrow R^{-1} = \begin{pmatrix} 2 & 7 & 2 \\ 3 & 1 & 10 \\ 6 & 3 & 0 \end{pmatrix}, \quad J = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 8 \end{pmatrix}$$

$\begin{matrix} 1 & 2 & 8 \\ 4 & 7 & 10 \\ 1 & 1 & 1 \end{matrix}$	$\begin{matrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$	$\begin{matrix} 2 & 7 & 2 \\ 3 & 1 & 10 \\ 6 & 3 & 0 \end{matrix}$
$\begin{matrix} 1 & 2 & 8 \\ 4 & 7 & 10 \\ 1 & 1 & 1 \end{matrix}$	$\begin{matrix} 0 & 2 & 8 \\ 0 & 7 & 10 \\ 0 & 1 & 1 \end{matrix}$	$\begin{matrix} 10 & 4 & 9 \\ 4 & 4 & 4 \\ 9 & 4 & 10 \end{matrix}$

$$A^{1000} = \begin{pmatrix} 10 & 4 & 9 \\ 4 & 4 & 4 \\ 9 & 4 & 10 \end{pmatrix}$$