

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 4 & 1 \\ 2 & 3 & 3 \end{vmatrix} = (-1)^{2+1} \cdot 4 \cdot \begin{vmatrix} 2 & 3 \\ 3 & 3 \end{vmatrix} +$$

$$(-1)^{2+2} \cdot 4 \cdot \begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix} +$$

$$(-1)^{2+3} \cdot 1 \cdot \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 2 + 3 + 1 = 1$$

nach \mathbb{Z}_5

rechte 2. rachen

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 4 & 1 \\ 2 & 3 & 3 \end{vmatrix} = + 3 \cdot \begin{vmatrix} 4 & 4 \\ 2 & 3 \end{vmatrix} - 1 \cdot \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} + 3 \cdot \begin{vmatrix} 1 & 2 \\ 4 & 4 \end{vmatrix} = 2 + 1 + 3 = 1$$

rechte 3. spalten

$$\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 2 & \\ 5 & 6 & 0 & 5 & 6 & \\ 0 & 0 & 8 & 0 & 0 & \end{array}$$

$$\begin{array}{ccc|ccc} 0 & 1 & 2 & 0 & 1 & \\ 0 & 5 & 6 & 0 & 5 & \\ 7 & 0 & 0 & 7 & 0 & \end{array}$$

$$\rightarrow \begin{vmatrix} 0 & 1 & 2 & 0 \\ 3 & 0 & 0 & 4 \\ 0 & 5 & 6 & 0 \\ 7 & 0 & 0 & 8 \end{vmatrix} = -3 \cdot \begin{vmatrix} 1 & 2 & 0 \\ 5 & 6 & 0 \\ 0 & 0 & 8 \end{vmatrix} + 4 \cdot \begin{vmatrix} 0 & 1 & 2 \\ 0 & 5 & 6 \\ 7 & 0 & 0 \end{vmatrix} = \underline{\underline{-16}}$$

$$\begin{array}{r} 48 - 80 \\ - 32 \\ + 96 \end{array}$$

$$\begin{array}{r} 42 - 70 \\ - 28 \cdot 4 = -112 \end{array}$$

$$\begin{vmatrix} 0 & 1 & 2 & 0 \\ 3 & 100 & 100 & 4 \\ 0 & 5 & 6 & 0 \\ 7 & 100 & 100 & 8 \end{vmatrix} = -3 \cdot \begin{vmatrix} 1 & 2 & 0 \\ 5 & 6 & 0 \\ 100 & 100 & 8 \end{vmatrix} + 100 \cdot \begin{vmatrix} 0 & 2 & 0 \\ 0 & 6 & 0 \\ 7 & 100 & 8 \end{vmatrix} \\
 - 100 \cdot \begin{vmatrix} 0 & 1 & 0 \\ 0 & 5 & 0 \\ 7 & 100 & 8 \end{vmatrix} + 4 \cdot \begin{vmatrix} 0 & 1 & 2 \\ 0 & 5 & 6 \\ 7 & 100 & 100 \end{vmatrix} = -16$$

" 0

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 0 & 3 \\ 3 & 0 & 1 & 2 \\ 4 & 3 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -5 \\ 0 & -6 & -8 & -10 \\ 0 & -5 & -10 & -15 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -5 \\ 0 & 0 & 4 & 0 \\ 0 & 1 & 2 & 3 \end{vmatrix} \therefore \\
 = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 4 & 0 \\ 0 & 1 & 2 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{vmatrix} = 16 \cdot 4 = 64$$

$$\begin{vmatrix} -1 & -2 & 3 & -1 \\ 2 & 4 & -3 & 1 \\ 1 & 2 & -2 & -1 \\ -2 & -1 & 1 & -2 \end{vmatrix} = \begin{vmatrix} -1 & -2 & 3 & -1 & -1 & -2 & 3 & -1 \\ 0 & 0 & 3 & -1 & 0 & 0 & 0 & 5 \\ 0 & 0 & 1 & -2 & 0 & 0 & 1 & -2 \\ 0 & 3 & -5 & 0 & 0 & 3 & -5 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & -2 & 3 & -1 \\ 0 & 3 & -5 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 5 \end{vmatrix}$$

$$-15 = 15$$

$$\rightarrow \begin{vmatrix} 1 & -1 & 1 & 1 & 3 \\ 0 & 2 & 1 & 0 & -1 \\ 2 & -1 & 1 & -3 & 0 \\ 0 & 0 & -1 & 2 & 1 \\ 3 & 2 & 0 & 1 & 0 \end{vmatrix} = -1 \cdot \begin{vmatrix} 1 & -1 & 1 & 3 \\ 0 & 2 & 0 & -1 \\ 2 & -1 & -3 & 0 \\ 3 & 2 & 1 & 0 \end{vmatrix}$$

-43
"

$$+ 2 \cdot \begin{vmatrix} 1 & -1 & 1 & 3 \\ 0 & 2 & 1 & -1 \\ 2 & -1 & 1 & 0 \\ 3 & 2 & 0 & 0 \end{vmatrix} - 63$$

$$- \begin{vmatrix} 1 & -1 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 2 & -1 & 1 & -3 \\ 3 & 2 & 0 & 1 \end{vmatrix}$$

$$\begin{pmatrix} 1 & -1 & 1 & 3 \\ 0 & 2 & 0 & -1 \\ 0 & 1 & -5 & -6 \\ 0 & 5 & -2 & -9 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 & 3 \\ 0 & 0 & 10 & 11 \\ 0 & 1 & -5 & -6 \\ 0 & 0 & 23 & 21 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 & 3 \\ 0 & 0 & 10 & 11 \\ 0 & 1 & -5 & -6 \\ 0 & 0 & 13 & 10 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & -1 & 1 & 3 \\ 0 & 0 & 10 & 11 \\ 0 & 1 & -5 & -6 \\ 0 & 0 & 3 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 & 3 \\ 0 & 0 & 0 & \frac{43}{3} \\ 0 & 1 & -5 & -6 \\ 0 & 0 & 3 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 & 3 \\ 0 & 1 & -5 & -6 \\ 0 & 0 & 3 & -1 \\ 0 & 0 & 0 & \frac{43}{3} \end{pmatrix}$$

$$(1 \ 3 \ 4 \ 2) \Rightarrow \text{sgn} = -1$$

$$\begin{pmatrix} 1 & -1 & 1 & 3 \\ 0 & 2 & 1 & -1 \\ 2 & -1 & 1 & 0 \\ 3 & 2 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 & 3 \\ 0 & 2 & 1 & -1 \\ 0 & 1 & -1 & -6 \\ 0 & 5 & -3 & -9 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 & 3 \\ 0 & 1 & -1 & -6 \\ 0 & 0 & 3 & 11 \\ 0 & 0 & 2 & 21 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & -1 & 1 & 3 \\ 0 & 1 & -1 & -6 \\ 0 & 0 & 0 & -\frac{63}{2} \\ 0 & 0 & 2 & 11 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 & 3 \\ 0 & 1 & -1 & -6 \\ 0 & 0 & 2 & 11 \\ 0 & 0 & 0 & -\frac{63}{2} \end{pmatrix} = +63 \quad -\frac{3}{2} \cdot \frac{21}{1} = -\frac{63}{2}$$

$$\begin{pmatrix} 1 & -1 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 2 & -1 & 1 & -3 \\ 3 & 2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & -1 & -5 \\ 0 & 5 & -3 & -2 \end{pmatrix}$$